NCERT Class 12 Maths

Chapter-wise Solutions

Chapter 12

Linear Programming

Complete Step-by-Step NCERT Solutions

In this document — Complete Chapter

All 31 questions of Chapter 12, fully solved.

For Class 12 CBSE / NCERT students

Every question solved with clear, exam-ready working

Prepared as a print-friendly, exam-preparation resource

Chapter 12: Linear Programming — Overview

A **Linear Programming Problem (LPP)** seeks the optimal (maximum or minimum) value of a linear function, called the **objective function** $Z = ax + by$, subject to a set of linear inequalities called **constraints**. The variables x, y are called **decision variables** and are always non-negative ($x \geq 0$, $y \geq 0$).

Key Concepts Used

Feasible region: the common region satisfying all constraints (including $x, y \geq 0$). Any point in it is a **feasible solution**.

Corner Point Method (used to solve every problem below):

1. Find the feasible region and its corner points (vertices), either graphically or by solving intersecting constraint lines simultaneously.
2. Evaluate $Z = ax + by$ at each corner point.
3. **Bounded region:** the largest value of Z is the maximum and the smallest is the minimum.
4. **Unbounded region:** a candidate maximum/minimum found this way is genuine *only if* the open half-plane $ax + by > M$ (or $< m$) has no point in common with the feasible region; otherwise no such optimal value exists.
5. If two corner points give the same optimal value of Z, every point of the line segment joining them is also optimal (infinitely many solutions).

These rules are applied identically to every problem that follows — only the formulation of constraints changes from question to question.

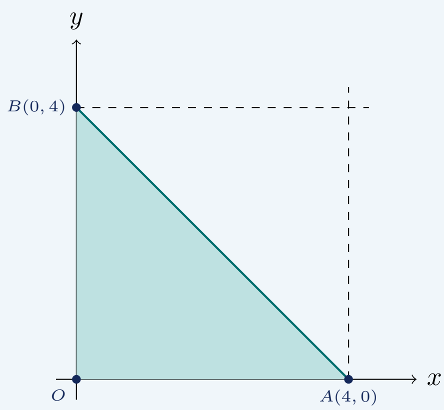
1 Exercise 12.1

Solve the following Linear Programming Problems graphically.

Question 1

Question 1. Maximise $Z = 3x + 4y$ subject to $x + y \leq 4,\ x \geq 0,\ y \geq 0$.

Solution:



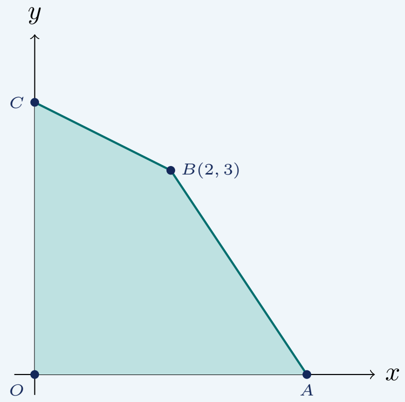
The feasible region is bounded by $x + y \leq 4$ in the first quadrant. Its corner points are $O(0, 0)$, $A(4, 0)$ and $B(0, 4)$.

Corner Point	Value of $Z = 3x + 4y$
$O(0,0)$	0
$A(4,0)$	12
$B(0,4)$	16 ← max

Answer: Maximum value of Z is 16 at $B(0, 4)$.

Question 2

Question 2. Minimise $Z = -3x + 4y$ subject to $x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0$.
Solution:



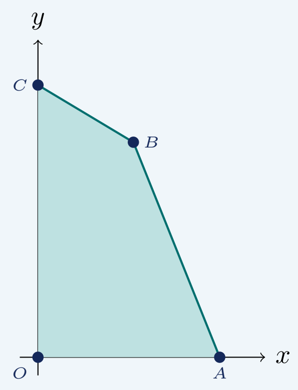
Solving $x + 2y = 8$ and $3x + 2y = 12$ simultaneously gives $x = 2, y = 3$. The corner points are $O(0,0)$, $A(4,0)$, $B(2,3)$ and $C(0,4)$.

Corner Point	Value of $Z = -3x + 4y$
$O(0,0)$	0
$A(4,0)$	$-12 \leftarrow$ min
$B(2,3)$	6
$C(0,4)$	16

Answer: Minimum value of Z is -12 at $A(4,0)$.

Question 3

Question 3. Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15,\ 5x + 2y \leq 10,\ x \geq 0,\ y \geq 0$.
Solution:



Solving $3x+5y=15$ and $5x+2y=10$ simultaneously: $x=\dfrac{20}{19}$, $y=\dfrac{45}{19}$. Corner points: $O(0,0)$, $A(2,0)$, $B\left(\frac{20}{19},\frac{45}{19}\right)$, $C(0,3)$.

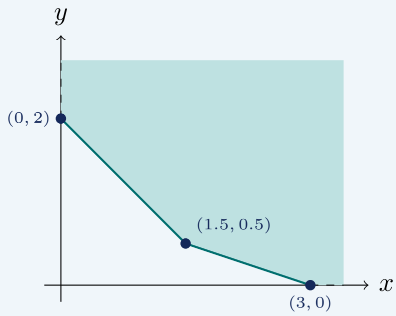
Corner Point	Value of $Z = 5x + 3y$
$O(0,0)$	0
$A(2,0)$	10
$B\left(\frac{20}{19},\frac{45}{19}\right)$	$\frac{235}{19} \approx 12.37 \leftarrow$ max
$C(0,3)$	9

Answer: Maximum value of Z is $\dfrac{235}{19}$ at $\left(\dfrac{20}{19},\dfrac{45}{19}\right)$.

Question 4

Question 4. Minimise $Z = 3x + 5y$ such that $x + 3y \geq 3,\ x + y \geq 2,\ x, y \geq 0$.

Solution:



This is an unbounded region (both constraints are $\geq$ type). Solving $x+3y = 3$ and $x+y = 2$ simultaneously: $x = 1.5$, $y = 0.5$. Corner points: $(3, 0)$, $\left(\frac{3}{2}, \frac{1}{2}\right)$, $(0, 2)$.

Corner Point	Value of $Z = 3x + 5y$
$(3, 0)$	9
$\left(\frac{3}{2}, \frac{1}{2}\right)$	$7 \leftarrow$ min
$(0, 2)$	10

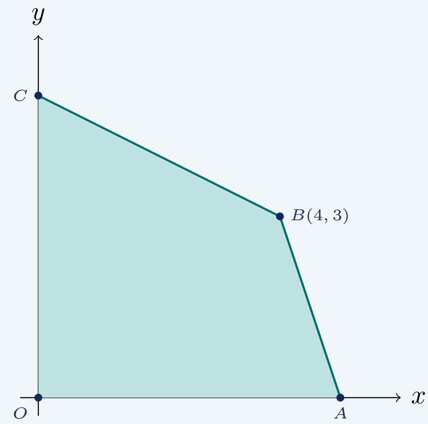
Since the region is unbounded, we check whether $3x + 5y < 7$ has any point common with the feasible region – it does not. So $Z = 7$ is genuinely the minimum.

Answer: Minimum value of Z is 7 at $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$.

Question 5

Question 5. Maximise $Z = 3x + 2y$ subject to $x + 2y \leq 10,\ 3x + y \leq 15,\ x, y \geq 0$.

Solution:



Solving $x + 2y = 10$ and $3x + y = 15$ simultaneously: $x = 4$, $y = 3$. Corner points: $O(0, 0)$, $A(5, 0)$, $B(4, 3)$, $C(0, 5)$.

Corner Point	Value of $Z = 3x + 2y$
$O(0, 0)$	0
$A(5, 0)$	15
$B(4, 3)$	18 $\leftarrow$ max
$C(0, 5)$	10

Answer: Maximum value of Z is 18 at $B(4, 3)$.

Question 6

Question 6. Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$. Show that the minimum of Z occurs at more than two points.

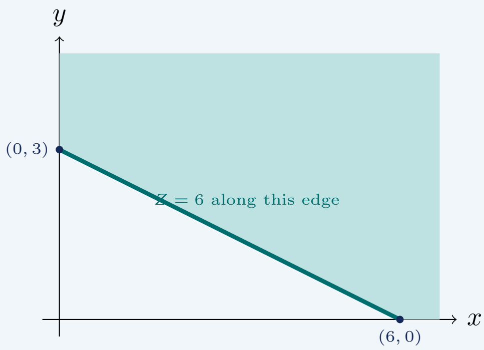
Solution:

Corner points of the (unbounded) feasible region are $(6, 0)$ and $(0, 3)$; solving $2x + y = 3$ and $x + 2y = 6$ together also gives $(0, 3)$.

$$Z(6, 0) = 6, \qquad Z(0, 3) = 6$$

Both corner points give $Z = 6$. Since $Z = x + 2y$ is a scalar multiple of the constraint $x + 2y \geq 6$ itself, $Z = 6$ along the *entire* segment joining $(6, 0)$ and $(0, 3)$ where $x + 2y = 6$ – not just at the two endpoints. Checking that $x + 2y < 6$ has no point common with the feasible region confirms this is indeed the minimum.

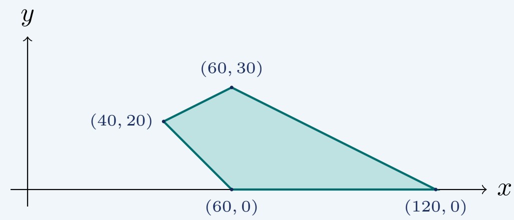
Answer: Minimum value of Z is 6, occurring at *every* point of the line segment joining $(6, 0)$ and $(0, 3)$ – i.e. at infinitely many points, not just two.



Question 7

Question 7. Minimise and Maximise $Z = 5x + 10y$ subject to $x + 2y \leq 120,\ x + y \geq 60,\ x - 2y \geq 0,\ x, y \geq 0$.

Solution:



Solving the constraint lines pairwise (keeping only points satisfying *all* constraints) gives a bounded quadrilateral feasible region with corner points $(60, 0)$, $(120, 0)$, $(60, 30)$, $(40, 20)$.

Corner Point	Value of $Z = 5x + 10y$
$(60, 0)$	$300 \leftarrow$ min
$(40, 20)$	400
$(60, 30)$	$600 \leftarrow$ max
$(120, 0)$	$600 \leftarrow$ max

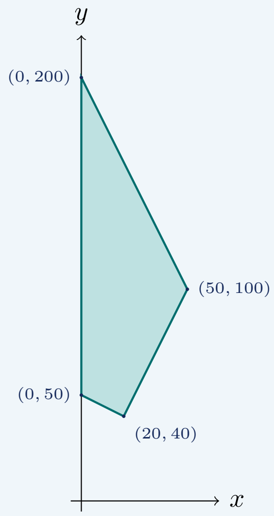
Since $(60, 30)$ and $(120, 0)$ give the same value, $Z = 600$ along the whole segment joining them.

Answer: Minimum value $= 300$ at $(60, 0)$. Maximum value $= 600$, occurring at every point of the segment joining $(60, 30)$ and $(120, 0)$.

Question 8

Question 8. Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$.

Solution:



The bounded feasible region is a quadrilateral with corner points $(20, 40)$, $(0, 50)$, $(0, 200)$, $(50, 100)$.

Corner Point	Value of $Z = x + 2y$
$(0, 50)$	$100 \leftarrow$ min
$(20, 40)$	$100 \leftarrow$ min
$(50, 100)$	250
$(0, 200)$	$400 \leftarrow$ max

Since $(0, 50)$ and $(20, 40)$ give the same value, $Z = 100$ along the whole segment joining them (this edge lies on $x + 2y = 100$, the same expression as Z).

Answer: Minimum value $= 100$, occurring at every point of the segment joining $(0, 50)$ and $(20, 40)$. Maximum value $= 400$ at $(0, 200)$.

Question 9

Question 9. Maximise $Z = -x + 2y$, subject to the constraints: $x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

Solution:

This region is unbounded, with corner points $(6, 0)$, $(4, 1)$, $(3, 2)$, and extending upward without bound along $x = 3$.

Corner Point	Value of $Z = -x + 2y$
$(6, 0)$	-6
$(4, 1)$	-2
$(3, 2)$	1

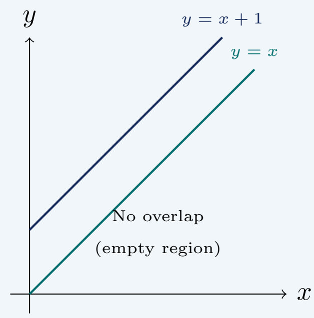
The region is unbounded and extends indefinitely upward along $x = 3$ (with $y \to \infty$), where $Z = -3+2y \to \infty$. So Z increases without any upper bound on this region.

Answer: Z has **no maximum value** – it can be made arbitrarily large on this unbounded region.

Question 10

Question 10. Maximise $Z = x + y$, subject to $x - y \leq -1,\ -x + y \leq 0,\ x, y \geq 0$.

Solution:



The constraint $x - y \leq -1$ requires $y \geq x + 1$, while $-x + y \leq 0$ requires $y \leq x$. These two conditions cannot hold simultaneously (we would need $x + 1 \leq y \leq x$, i.e. $x + 1 \leq x$, which is impossible).

Answer: There is **no feasible region** – the two constraints are mutually contradictory. Hence Z has no maximum value.

End of Exercise 12.1 — all 10 questions solved.

2 Exercise 12.2

Application problems: diet, manufacturing, and allocation problems, solved using the corner point method.

Question 1: Diet Problem

Question 1. Reshma wishes to mix two types of food P and Q so that the mixture contains at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B, while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Solution:

Let the mixture contain x kg of food P and y kg of food Q.

$$\text{Minimise } Z = 60x + 80y \quad \text{subject to} \quad 3x + 4y \geq 8,\ 5x + 2y \geq 11,\ x, y \geq 0$$

Solving $3x + 4y = 8$ and $5x + 2y = 11$ simultaneously gives $x = 2,\ y = \frac{1}{2}$.

Corner Point	Value of $Z = 60x + 80y$
$\left(\frac{8}{3}, 0\right)$	$160 \leftarrow$ min
$\left(2, \frac{1}{2}\right)$	$160 \leftarrow$ min
$(0, 5.5)$	440

Since both $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$ give $Z = 160$, and this edge lies on $3x + 4y = 8$ (proportional to Z), the minimum cost occurs along the whole segment joining them. Checking $60x + 80y < 160$ confirms no lower value is feasible.

Answer: Minimum cost is Rs 160, attained at every point of the segment joining $\left(\dfrac{8}{3}, 0\right)$ and $\left(2, \dfrac{1}{2}\right)$.

Question 2: Manufacturing Problem

Question 2. One kind of cake requires 200 g flour and 25 g fat, another kind requires 100 g flour and 50 g fat. Find the maximum number of cakes that can be made from 5 kg flour and 1 kg fat, assuming no shortage of the other ingredients.

Solution:

Let x = number of cakes of the first kind, y = number of the second kind.

$$\text{Maximise } Z = x + y \quad \text{s.t.} \quad 200x + 100y \leq 5000,\ 25x + 50y \leq 1000,\ x, y \geq 0$$

i.e. $2x + y \leq 50$ and $x + 2y \leq 40$. Solving simultaneously: $x = 20,\ y = 10$.

Corner Point	Value of $Z = x + y$
$O(0, 0)$	0
$(25, 0)$	25
$(20, 10)$	$30 \leftarrow$ max
$(0, 20)$	20

Answer: Maximum number of cakes = 30: 20 cakes of the first kind and 10 of the second kind.

Question 3: Factory Production

Question 3. A factory makes tennis rackets and cricket bats. A racket takes 1.5 hours machine time and 3 hours craftsman's time; a bat takes 3 hours machine time and 1 hour craftsman's time. The factory has at most 42 hours of machine time and 24 hours of craftsman's time per day.
(i) What number of rackets and bats must be made to work at full capacity?
(ii) If profit is Rs 20 per racket and Rs 10 per bat, find the maximum profit.

Solution:
Let x = number of rackets, y = number of bats.
(i) Working at full capacity means both resources are fully used:

$$1.5x + 3y = 42, \qquad 3x + y = 24$$

From the second equation, $y = 24 - 3x$. Substituting: $1.5x + 3(24 - 3x) = 42 \Rightarrow -7.5x = -30 \Rightarrow x = 4,\ y = 12$.

Answer: (i) 4 rackets and 12 bats must be made to work at full capacity.

(ii) Maximise $Z = 20x + 10y$ subject to $1.5x + 3y \leq 42,\ 3x + y \leq 24,\ x, y \geq 0$.

Corner Point	Value of $Z = 20x + 10y$
$O(0, 0)$	0
$(8, 0)$	160
$(4, 12)$	$200 \leftarrow$ max
$(0, 14)$	140

Answer: (ii) Maximum profit is Rs 200, at $x = 4$ rackets, $y = 12$ bats – confirming full-capacity production is also most profitable.

Question 4: Nuts and Bolts

Question 4. A manufacturer produces nuts and bolts. A package of nuts needs 1 hour on machine A and 3 hours on machine B; a package of bolts needs 3 hours on machine A and 1 hour on machine B. Profit is Rs 17.50/package of nuts and Rs 7/package of bolts. Each machine is available at most 12 hours a day. Maximise profit.

Solution:
Let x = packages of nuts, y = packages of bolts.

$$\text{Maximise } Z = 17.5x + 7y \quad \text{s.t.} \quad x + 3y \leq 12,\ 3x + y \leq 12,\ x, y \geq 0$$

Solving simultaneously: $x = 3,\ y = 3$.

Corner Point	Value of $Z = 17.5x + 7y$
$O(0, 0)$	0
$(4, 0)$	70
$(3, 3)$	$73.50 \leftarrow$ max
$(0, 4)$	28

Answer: Maximum profit is Rs 73.50, by producing 3 packages of nuts and 3 packages of bolts daily.

Question 5: Screws Manufacturing

Question 5. A factory manufactures screws A and B. Screw A needs 4 min on an automatic and 6 min on a hand-operated machine; screw B needs 6 min automatic and 3 min hand-operated. Each machine is available at most 4 hours a day. Profit is Rs 7 per package of A and Rs 10 per package of B. Maximise profit.

Solution:

Let x = packages of screw A, y = packages of screw B (each machine available for 240 min/day).

$$\text{Maximise } Z = 7x + 10y \quad \text{s.t.} \quad 2x + 3y \leq 120,\ 2x + y \leq 80,\ x, y \geq 0$$

Solving simultaneously: $x = 30,\ y = 20$.

Corner Point	Value of $Z = 7x + 10y$
$O(0,0)$	0
$(40,0)$	280
$(30,20)$	410 $\leftarrow$ max
$(0,40)$	400

Answer: Maximum profit is Rs 410, by producing 30 packages of screw A and 20 packages of screw B daily.

Question 6: Cottage Industry

Question 6. A cottage industry manufactures pedestal lamps and wooden shades. A lamp needs 2 hours grinding/cutting and 3 hours on the sprayer; a shade needs 1 hour grinding/cutting and 2 hours sprayer. The sprayer is available at most 20 hours, and the grinding/cutting machine at most 12 hours a day. Profit is Rs 5 per lamp and Rs 3 per shade. Maximise profit.

Solution:

Let x = number of lamps, y = number of shades.

$$\text{Maximise } Z = 5x + 3y \quad \text{s.t.} \quad 2x + y \leq 12,\ 3x + 2y \leq 20,\ x, y \geq 0$$

Solving simultaneously: $x = 4,\ y = 4$.

Corner Point	Value of $Z = 5x + 3y$
$O(0,0)$	0
$(6,0)$	30
$(4,4)$	32 $\leftarrow$ max
$(0,10)$	30

Answer: Maximum profit is Rs 32, by producing 4 lamps and 4 shades daily.

Question 7: Souvenirs

Question 7. A company manufactures two types of plywood souvenirs. Type A needs 5 min cutting and 10 min assembling; type B needs 8 min cutting and 8 min assembling. There are 3 hours 20 minutes (200 min) available for cutting, and 4 hours (240 min) for assembling. Profit is Rs 5 per type A and Rs 6 per type B souvenir. Maximise profit.

Solution:

Let x = souvenirs of type A, y = souvenirs of type B.

$$\text{Maximise } Z = 5x + 6y \quad \text{s.t.} \quad 5x + 8y \leq 200,\ 5x + 4y \leq 120,\ x, y \geq 0$$

Solving simultaneously: $x = 8,\ y = 20$.

Corner Point	Value of $Z = 5x + 6y$
$O(0,0)$	0
$(24,0)$	120
$(8,20)$	$160 \leftarrow$ max
$(0,25)$	150

Answer: Maximum profit is Rs 160, by making 8 souvenirs of type A and 20 of type B.

Question 8: Computer Sales

Question 8. A merchant plans to sell desktop (Rs 25,000) and portable (Rs 40,000) computers. Total monthly demand will not exceed 250 units, and he will not invest more than Rs 70 lakh. Profit is Rs 4500 per desktop and Rs 5000 per portable. Maximise profit.

Solution:

Let x = desktop units, y = portable units.

$$\text{Maximise } Z = 4500x + 5000y \quad \text{s.t.} \quad x + y \leq 250,\ 5x + 8y \leq 1400,\ x, y \geq 0$$

Solving simultaneously: $x = 200,\ y = 50$.

Corner Point	Value of $Z = 4500x + 5000y$
$O(0,0)$	0
$(250,0)$	11,25,000
$(200,50)$	$11{,}50{,}000 \leftarrow$ max
$(0,175)$	8,75,000

Answer: Maximum profit is Rs 11,50,000, by stocking 200 desktop and 50 portable computers.

Question 9: Diet – Minimum Cost

Question 9. A diet must contain at least 80 units of vitamin A and 100 units of minerals. Food F1 costs Rs 4/unit (3 units vitamin A, 4 units minerals per unit); Food F2 costs Rs 6/unit (6 units vitamin A, 3 units minerals per unit). Formulate as an LPP and find the minimum cost.

Solution:

Let x = units of F1, y = units of F2.

$$\text{Minimise } Z = 4x + 6y \quad \text{s.t.} \quad 3x + 6y \geq 80,\ 4x + 3y \geq 100,\ x, y \geq 0$$

Solving simultaneously: $x = 24,\ y = \frac{4}{3}$.

Corner Point	Value of $Z = 4x + 6y$
$\left(\frac{80}{3}, 0\right)$	≈ 106.67
$\left(24, \frac{4}{3}\right)$	$104 \leftarrow$ min
$\left(0, \frac{100}{3}\right)$	200

The region is unbounded; checking $4x + 6y < 104$ confirms no feasible point exists there.

Answer: Minimum cost is Rs 104, using 24 units of F1 and $\dfrac{4}{3}$ units of F2.

Question 10: Fertilisers

Question 10. Fertiliser F1 has 10% nitrogen and 6% phosphoric acid; F2 has 5% nitrogen and 10% phosphoric acid. A farmer needs at least 14 kg nitrogen and 14 kg phosphoric acid. F1 costs Rs 6/kg, F2 costs Rs 5/kg. Minimise cost.

Solution:

Let x kg of F1 and y kg of F2 be used.

$$\text{Minimise } Z = 6x + 5y \quad \text{s.t.} \quad 2x + y \geq 280,\ 3x + 5y \geq 700,\ x, y \geq 0$$

(from $10x+5y \geq 1400$ and $6x+10y \geq 1400$, simplified). Solving simultaneously: $x = 100,\ y = 80$.

Corner Point	Value of $Z = 6x + 5y$
$\left(\frac{700}{3}, 0\right)$	1400
$(100, 80)$	1000 ← min
$(0, 280)$	1400

Answer: Minimum cost is Rs 1000, using 100 kg of F1 and 80 kg of F2.

Question 11: MCQ

Question 11. The corner points of the feasible region determined by the system of linear inequalities $2x + y \leq 10,\ x + 3y \leq 15,\ x, y \geq 0$ are $(0, 0), (5, 0), (3, 4)$ and $(0, 5)$. Let $Z = px + qy$ where $p, q > 0$. The condition on p and q so that the maximum of Z occurs at *both* $(3, 4)$ and $(5, 0)$ is:

(A) $p = q$ (B) $p = 2q$ (C) $p = 3q$ (D) $q = 3p$

Solution:

For the maximum to occur at both corner points, Z must take the same value at each:

$$Z(3, 4) = Z(5, 0) \ \Rightarrow\ 3p + 4q = 5p \ \Rightarrow\ 4q = 2p \ \Rightarrow\ p = 2q$$

This matches option (B).

Answer: Option (B) is correct: $p = 2q$.

End of Exercise 12.2 — all 11 questions solved.

3 Miscellaneous Exercise

Question 1

Question 1. Refer to Example 9 (food P and Q, vitamin diet problem). How many packets of each food should be used to *maximise* the amount of vitamin A in the diet, given the additional constraints $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$? What is the maximum amount of vitamin A?

Solution:

Let the diet contain x packets of food P and y packets of food Q.

$$\text{Maximise } Z = 6x + 3y \quad \text{s.t.} \quad 4x + y \geq 80,\ x + 5y \geq 115,\ 3x + 2y \leq 150,\ x, y \geq 0$$

Solving the constraint lines pairwise gives corner points $(15, 20)$, $(40, 15)$, $(2, 72)$.

Corner Point	Value of $Z = 6x + 3y$
$(15, 20)$	150
$(40, 15)$	$285 \leftarrow$ max
$(2, 72)$	228

Answer: Maximum vitamin A is 285 units, using 40 packets of food P and 15 packets of food Q.

Question 2

Question 2. A farmer mixes brands P and Q of cattle feed. Brand P (Rs 250/bag) contains 3 units of nutrient A, 2.5 units of B, 2 units of C. Brand Q (Rs 200/bag) contains 1.5 units of A, 11.25 units of B, 3 units of C. Minimum requirements of A, B, C are 18, 45, 24 units. Minimise cost.

Solution:

Let $x =$ bags of brand P, $y =$ bags of brand Q.

$$\text{Minimise } Z = 250x + 200y \quad \text{s.t.} \quad 3x + 1.5y \geq 18,\ 2.5x + 11.25y \geq 45,\ 2x + 3y \geq 24,\ x, y \geq 0$$

Corner points: $(18, 0)$, $(9, 2)$, $(3, 6)$, $(0, 12)$.

Corner Point	Value of $Z = 250x + 200y$
$(18, 0)$	4500
$(9, 2)$	2650
$(3, 6)$	$1950 \leftarrow$ min
$(0, 12)$	2400

The region is unbounded; checking $250x + 200y < 1950$ confirms no feasible point exists there.

Answer: Minimum cost is Rs 1950, using 3 bags of brand P and 6 bags of brand Q.

Question 3

Question 3. A dietician mixes foods X and Y so the mixture has at least 10 units of vitamin A, 12 units of vitamin B, and 8 units of vitamin C. Food X (Rs 16/kg) has 1, 2, 3 units/kg of vitamins A, B, C respectively; Food Y (Rs 20/kg) has 2, 2, 1 units/kg. Find the least cost.

Solution:

Let x kg of X and y kg of Y be mixed.

$$\text{Minimise } Z = 16x + 20y \quad \text{s.t.} \quad x + 2y \geq 10,\ x + y \geq 6,\ 3x + y \geq 8,\ x, y \geq 0$$

Corner points: $(10, 0)$, $(2, 4)$, $(1, 5)$, $(0, 8)$.

Corner Point	Value of $Z = 16x + 20y$
$(10, 0)$	160
$(2, 4)$	$112 \leftarrow$ min
$(1, 5)$	116
$(0, 8)$	160

Answer: Minimum cost is Rs 112, using 2 kg of food X and 4 kg of food Y.

Question 4

Question 4. A manufacturer makes toys A and B using three machines. Toy A needs 12, 18, 6 minutes on machines I, II, III; toy B needs 6, 0, 9 minutes. Each machine is available for at most 6 hours (360 min) a day. Profit is Rs 7.50 on A and Rs 5 on B. Show that 15 toys of A and 30 of B give maximum profit.
Solution:
Let x = toys of type A, y = toys of type B.

$$\text{Maximise } Z = 7.5x + 5y \quad \text{s.t.} \quad 2x + y \leq 60,\ x \leq 20,\ 2x + 3y \leq 120,\ x, y \geq 0$$

(from $12x + 6y \leq 360$, $18x \leq 360$, $6x + 9y \leq 360$). Corner points: $O(0, 0)$, $(20, 0)$, $(20, 20)$, $(15, 30)$, $(0, 40)$.

Corner Point	Value of $Z = 7.5x + 5y$
$O(0, 0)$	0
$(20, 0)$	150
$(20, 20)$	250
$(15, 30)$	$262.50 \leftarrow$ max
$(0, 40)$	200

Answer: Maximum profit Rs 262.50 occurs at $(15, 30)$ – confirming 15 toys of type A and 30 of type B should be made.

Question 5

Question 5. An aeroplane carries at most 200 passengers. Profit is Rs 1000 per executive ticket and Rs 600 per economy ticket. At least 20 executive seats are reserved, and economy passengers must be at least 4 times the executive passengers. Maximise profit.
Solution:
Let x = executive tickets, y = economy tickets.

$$\text{Maximise } Z = 1000x + 600y \quad \text{s.t.} \quad x + y \leq 200,\ x \geq 20,\ y \geq 4x,\ x, y \geq 0$$

Corner points: $(20, 80)$, $(40, 160)$, $(20, 180)$.

Corner Point	Value of $Z = 1000x + 600y$
$(20, 80)$	68,000
$(40, 160)$	1,36,000 ← max
$(20, 180)$	1,28,000

Answer: Maximum profit is Rs 1,36,000, by selling 40 executive and 160 economy tickets.

Question 6: Transportation Problem

Question 6. Godowns A (100 quintals) and B (50 quintals) supply ration shops D, E, F requiring 60, 50, 40 quintals. Transport cost per quintal: A→D = Rs6, A→E = Rs3, A→F = Rs2.50; B→D = Rs4, B→E = Rs2, B→F = Rs3. Minimise transportation cost.

Solution:

Let x, y quintals go from A to D, E respectively; then A→F $= 100 - x - y$, and correspondingly B→D$= 60 - x$, B→E$= 50 - y$, B→F$= x + y - 60$.

$$Z = 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60) = 2.5x + 1.5y + 410$$

subject to $x \leq 60,\ y \leq 50,\ 60 \leq x + y \leq 100,\ x, y \geq 0$. Corner points: $(60, 0)$, $(60, 40)$, $(50, 50)$, $(10, 50)$.

Corner Point	Value of $Z = 2.5x + 1.5y + 410$
$(60, 0)$	560
$(60, 40)$	620
$(50, 50)$	610
$(10, 50)$	510 ← min

Answer: Minimum cost is Rs 510: transport 10, 50, 40 quintals from A to D, E, F respectively, and 50, 0, 0 quintals from B to D, E, F respectively.

Question 7: Transportation Problem

Question 7. Oil depots A (7000 L) and B (4000 L) supply petrol pumps D, E, F needing 4500 L, 3000 L, 3500 L. Distances (km): A–D=7, A–E=6, A–F=3; B–D=3, B–E=4, B–F=2. Cost of transporting 10 L over 1 km is Re 1. Minimise transportation cost.

Solution:

Let x, y litres go from A to D, E; then A→F$= 7000 - x - y$, B→D$= 4500 - x$, B→E$= 3000 - y$, B→F$= x + y - 3500$.

$$Z = 0.1\,[7x + 6y + 3(7000 - x - y) + 3(4500 - x) + 4(3000 - y) + 2(x + y - 3500)] = 0.3x+0.1y+3950$$

subject to $x \leq 4500,\ y \leq 3000,\ 3500 \leq x + y \leq 7000,\ x, y \geq 0$. Corner points: $(3500, 0)$, $(4500, 0)$, $(4500, 2500)$, $(4000, 3000)$, $(500, 3000)$.

Corner Point	Value of $Z = 0.3x + 0.1y + 3950$
$(3500, 0)$	5000
$(4500, 0)$	5300
$(4500, 2500)$	5550
$(4000, 3000)$	5450
$(500, 3000)$	4400 ← min

Answer: Minimum cost is Rs 4400: from depot A supply 500 L, 3000 L, 3500 L to D, E, F; from depot B supply 4000 L, 0 L, 0 L to D, E, F.

Question 8: Fertiliser – Minimise Nitrogen

Question 8. A fruit grower uses fertiliser brands P and Q. Brand P has 3 kg nitrogen, 1 kg phosphoric acid, 3 kg potash, 1.5 kg chlorine per bag; brand Q has 3.5, 2, 1.5, 2 kg respectively. The garden needs at least 240 kg phosphoric acid, at least 270 kg potash, and at most 310 kg chlorine. Minimise the nitrogen added.

Solution:

Let x = bags of P, y = bags of Q.

$$\text{Minimise } Z = 3x + 3.5y \quad \text{s.t.} \quad x + 2y \geq 240,\ 3x + 1.5y \geq 270,\ 1.5x + 2y \leq 310,\ x, y \geq 0$$

This gives a bounded triangular feasible region with corner points $(40, 100)$, $(20, 140)$, $(140, 50)$.

Corner Point	Value of $Z = 3x + 3.5y$
$(40, 100)$	$470 \leftarrow$ min
$(20, 140)$	550
$(140, 50)$	595

Answer: Minimum nitrogen added is 470 kg, using 40 bags of brand P and 100 bags of brand Q.

Question 9: Fertiliser – Maximise Nitrogen

Question 9. Refer to Question 8. If the grower instead wants to *maximise* the nitrogen added, how many bags of each brand should be used? What is the maximum amount of nitrogen?

Solution:

Using the same feasible region and corner points as Question 8, we now look for the largest value of $Z = 3x + 3.5y$:

Corner Point	Value of $Z = 3x + 3.5y$
$(40, 100)$	470
$(20, 140)$	550
$(140, 50)$	$595 \leftarrow$ max

Answer: Maximum nitrogen added is 595 kg, using 140 bags of brand P and 50 bags of brand Q.

Question 10: Toy Manufacturing

Question 10. A toy company makes dolls A and B. Combined output should not exceed 1200 dolls/day. Demand for B is at most half that of A. Production of A can exceed 3 times that of B by at most 600 units. Profit is Rs 12 per doll A and Rs 16 per doll B. Maximise profit.

Solution:

Let x = dolls of type A, y = dolls of type B.

$$\text{Maximise } Z = 12x + 16y \quad \text{s.t.} \quad x + y \leq 1200,\ y \leq \frac{x}{2}\ (x \geq 2y),\ x - 3y \leq 600,\ x, y \geq 0$$

This gives a bounded quadrilateral feasible region with corner points $(0, 0)$, $(600, 0)$, $(1050, 150)$,

$(800, 400)$.

Corner Point	Value of $Z = 12x + 16y$
$(0, 0)$	0
$(600, 0)$	7200
$(1050, 150)$	15,000
$(800, 400)$	16,000 $\leftarrow$ max

Answer: Maximum profit is Rs 16,000, by producing 800 dolls of type A and 400 dolls of type B daily.

End of Miscellaneous Exercise — all 10 questions solved.

Chapter 12: Linear Programming — Complete

All 31 questions across Exercises 12.1, 12.2, and the Miscellaneous Exercise have been solved.