# NCERT Solutions

Class 12 Mathematics

## Chapter 13: Probability

Complete Step-by-Step Solutions

Exercises 13.1, 13.2, 13.3 & Miscellaneous Exercise

- All questions solved with detailed, step-by-step working
- Clear explanations of conditional probability, independence, and Bayes' theorem
- Ideal for CBSE Board exam preparation (rationalised 2026-27 syllabus)
- Print-friendly, distraction-free layout

## Quick Revision: Key Concepts & Formulas

- **Conditional probability:** $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$, provided $P(F) \neq 0$. It represents the probability of $E$ given that $F$ has already occurred.
- **Multiplication theorem:** $P(E \cap F) = P(F) \cdot P(E|F) = P(E) \cdot P(F|E)$. For three events: $P(E \cap F \cap G) = P(E) \cdot P(F|E) \cdot P(G|E \cap F)$.
- **Independent events:** $E$ and $F$ are independent if $P(E \cap F) = P(E) \cdot P(F)$. Equivalently, $P(E|F) = P(E)$ and $P(F|E) = P(F)$. If $E, F$ are independent, so are $E', F$; $E, F'$; and $E', F'$.
- **Mutually exclusive vs. independent:** Mutually exclusive means $P(E \cap F) = 0$; independent means $P(E \cap F) = P(E)P(F)$. Two events with nonzero probability cannot be both mutually exclusive and independent.
- **Partition of sample space:** Events $E_1, E_2, \ldots, E_n$ form a partition of $S$ if they are pairwise disjoint, exhaustive ($E_1 \cup E_2 \cup \cdots \cup E_n = S$), and each has nonzero probability.
- **Theorem of total probability:** If $E_1, \ldots, E_n$ partition $S$ and $A$ is any event, then

$$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A|E_i)$$

- **Bayes' theorem:** If $E_1, \ldots, E_n$ partition $S$ with $P(E_i) > 0$, and $A$ is any event with $P(A) > 0$, then

$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\, P(A|E_j)}$$

- **Common triggers:** "Given that" $\rightarrow$ conditional probability. "Independently" $\rightarrow$ multiply probabilities. "Given the observed effect, find the probability of a particular cause" $\rightarrow$ Bayes' theorem.

# 1 Exercise 13.1

### Question 1

Given that $E$ and $F$ are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E|F)$ and $P(F|E)$.

***Solution:***

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

### Question 2

Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

***Solution:***

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$$

### Question 3

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find:
(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(A \cup B)$

***Solution:*** **(i)** $P(A \cap B) = P(A) \cdot P(B|A) = 0.8 \times 0.4 = 0.32$
**(ii)** $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.32}{0.5} = 0.64$
**(iii)** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.5 - 0.32 = 0.98$

### Question 4

Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \dfrac{5}{13}$ and $P(A|B) = \dfrac{2}{5}$.

***Solution:*** From $2P(A) = \dfrac{5}{13}$: $P(A) = \dfrac{5}{26}$. Also $P(B) = \dfrac{5}{13}$.

$$P(A \cap B) = P(B) \cdot P(A|B) = \frac{5}{13} \times \frac{2}{5} = \frac{2}{13}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5}{26} + \frac{10}{26} - \frac{4}{26} = \frac{11}{26}$$

### Question 5

If $P(A) = \dfrac{6}{11}$, $P(B) = \dfrac{5}{11}$ and $P(A \cup B) = \dfrac{7}{11}$, find:
(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(B|A)$

***Solution:*** **(i)** $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \dfrac{6}{11} + \dfrac{5}{11} - \dfrac{7}{11} = \dfrac{4}{11}$
**(ii)** $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{4/11}{5/11} = \dfrac{4}{5}$
**(iii)** $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{4/11}{6/11} = \dfrac{2}{3}$

## Question 6

A coin is tossed three times, where:
(i) $E$: head on third toss, $F$: heads on first two tosses
(ii) $E$: at least two heads, $F$: at most two heads
(iii) $E$: at most two tails, $F$: at least one tail
Determine $P(E|F)$ in each case.

***Solution:*** Sample space $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$, so $n(S) = 8$.
**(i)** $F = \{HHH, HHT\}$, so $P(F) = \frac{2}{8}$. $E \cap F = \{HHH\}$, so $P(E \cap F) = \frac{1}{8}$.

$$P(E|F) = \frac{1/8}{2/8} = \frac{1}{2}$$

**(ii)** $E = \{HHH, HHT, HTH, THH\}$, $F = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$ (7 outcomes, all except HHH). $E \cap F = \{HHT, HTH, THH\}$, so $P(E \cap F) = \frac{3}{8}$, $P(F) = \frac{7}{8}$.

$$P(E|F) = \frac{3/8}{7/8} = \frac{3}{7}$$

**(iii)** $E$ (at most 2 tails) excludes only TTT, so $P(E) = \frac{7}{8}$. $F$ (at least 1 tail) excludes only HHH, so $P(F) = \frac{7}{8}$. $E \cap F$ excludes HHH and TTT, so $P(E \cap F) = \frac{6}{8} = \frac{3}{4}$.

$$P(E|F) = \frac{6/8}{7/8} = \frac{6}{7}$$

## Question 7

Two coins are tossed once, where:
(i) $E$: tail appears on one coin, $F$: one coin shows head
(ii) $E$: no tail appears, $F$: no head appears
Determine $P(E|F)$.

***Solution:*** Sample space $S = \{HH, HT, TH, TT\}$.
**(i)** $E = \{HT, TH\}$ (exactly one tail), $F = \{HT, TH\}$ (exactly one head) – these describe the same outcomes. $E \cap F = \{HT, TH\}$, so $P(E \cap F) = \frac{2}{4} = \frac{1}{2}$, $P(F) = \frac{2}{4} = \frac{1}{2}$.

$$P(E|F) = \frac{1/2}{1/2} = 1$$

**(ii)** $E = \{HH\}$, $F = \{TT\}$. $E \cap F = \emptyset$, so $P(E \cap F) = 0$ and $P(F) = \frac{1}{4}$.

$$P(E|F) = \frac{0}{1/4} = 0$$

## Question 8

A die is thrown three times. $E$: 4 appears on the third toss, $F$: 6 and 5 appear respectively on first two tosses. Find $P(E|F)$.

***Solution:*** $F = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$, so $P(F) = \frac{6}{216}$. $E \cap F = \{(6,5,4)\}$, so $P(E \cap F) = \frac{1}{216}$.

$$P(E|F) = \frac{1/216}{6/216} = \frac{1}{6}$$

## Question 9

Mother, father and son line up at random for a family picture. $E$: son on one end, $F$: father in middle. Find $P(E|F)$.

***Solution:*** Total arrangements of 3 people = 3! = 6: MFS, MSF, FMS, FSM, SMF, SFM (M=mother, F=father, S=son). $F$ (father in middle) $= \{MFS, SFM\}$, so $P(F) = \frac{2}{6}$. $E \cap F$ (father in middle AND son on an end) $= \{MFS, SFM\}$ – both already have son at an end, so $E \cap F = F$, giving $P(E \cap F) = \frac{2}{6}$.

$$P(E|F) = \frac{2/6}{2/6} = 1$$

## Question 10

A black and a red die are rolled.
(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

***Solution:*** **(a)** Let $F$: black die shows 5, so $F = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$, $P(F) = \frac{6}{36}$. Let $E$: sum $> 9$. $E \cap F = \{(5,5),(5,6)\}$ (sums 10, 11), so $P(E \cap F) = \frac{2}{36}$.

$$P(E|F) = \frac{2/36}{6/36} = \frac{1}{3}$$

**(b)** Let $F$: red die shows number less than 4, i.e., 1, 2, or 3, so $P(F) = \frac{18}{36}$. Let $E$: sum = 8. Valid (black, red) pairs with red $< 4$: red=2 $\to$ black=6; red=3 $\to$ black=5 (red=1 $\to$ black=7 is invalid). So $E \cap F = \{(6,2),(5,3)\}$, $P(E \cap F) = \frac{2}{36}$.

$$P(E|F) = \frac{2/36}{18/36} = \frac{1}{9}$$

## Question 11

A fair die is rolled. Consider events $E = \{1,3,5\}$, $F = \{2,3\}$, $G = \{2,3,4,5\}$. Find:
(i) $P(E|F)$ and $P(F|E)$ (ii) $P(E|G)$ and $P(G|E)$ (iii) $P((E \cup F)|G)$ and $P((E \cap F)|G)$

***Solution:*** $P(E) = \frac{3}{6} = \frac{1}{2}$, $P(F) = \frac{2}{6} = \frac{1}{3}$, $P(G) = \frac{4}{6} = \frac{2}{3}$.
**(i)** $E \cap F = \{3\}$, $P(E \cap F) = \frac{1}{6}$. $P(E|F) = \dfrac{1/6}{1/3} = \dfrac{1}{2}$, $\quad P(F|E) = \dfrac{1/6}{1/2} = \dfrac{1}{3}$
**(ii)** $E \cap G = \{3, 5\}$, $P(E \cap G) = \frac{2}{6} = \frac{1}{3}$. $P(E|G) = \dfrac{1/3}{2/3} = \dfrac{1}{2}$, $\quad P(G|E) = \dfrac{1/3}{1/2} = \dfrac{2}{3}$
**(iii)** $E \cup F = \{1, 2, 3, 5\}$, $(E \cup F) \cap G = \{2, 3, 5\}$, so $P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$.

$$P((E \cup F)|G) = \frac{1/2}{2/3} = \frac{3}{4}$$

$E \cap F = \{3\}$, $(E \cap F) \cap G = \{3\}$, so $P((E \cap F) \cap G) = \frac{1}{6}$.

$$P((E \cap F)|G) = \frac{1/6}{2/3} = \frac{1}{4}$$

## Question 12

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that:
(i) the youngest is a girl (ii) at least one is a girl?

***Solution:*** Sample space $S = \{BB, BG, GB, GG\}$ (first letter = older child).
**(i)** Let $F$: youngest is a girl $= \{BG, GG\}$, $P(F) = \frac{2}{4}$. Let $E$: both are girls $= \{GG\}$. $E \cap F = \{GG\}$, $P(E \cap F) = \frac{1}{4}$.

$$P(E|F) = \frac{1/4}{2/4} = \frac{1}{2}$$

**(ii)** Let $F$: at least one girl $= \{BG, GB, GG\}$, $P(F) = \frac{3}{4}$. $E \cap F = \{GG\}$, $P(E \cap F) = \frac{1}{4}$.

$$P(E|F) = \frac{1/4}{3/4} = \frac{1}{3}$$

## Question 13

An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions, and 400 difficult multiple choice questions. If a question is selected at random, what is the probability that it will be an easy question, given that it is a multiple choice question?

***Solution:*** Let $F$: question is multiple choice. Multiple choice questions $= 500 + 400 = 900$. Let $E$: question is easy. Easy multiple choice questions $= 500$.

$$P(E|F) = \frac{\text{easy MCQ}}{\text{total MCQ}} = \frac{500}{900} = \frac{5}{9}$$

## Question 14

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

***Solution:*** Let $F$: the two numbers are different. Number of outcomes with different numbers $= 36 - 6 = 30$ (excluding the 6 doublets), so $P(F) = \dfrac{30}{36}$. Let $E$: sum is 4. Outcomes: $(1,3),(3,1),(2,2)$. Since $(2,2)$ has equal numbers, it's excluded from $F$; so within $F$, $E \cap F = \{(1,3),(3,1)\}$, giving $P(E \cap F) = \frac{2}{36}$.

$$P(E|F) = \frac{2/36}{30/36} = \frac{2}{30} = \frac{1}{15}$$

**Question 15**

Consider the experiment of throwing a die. If a multiple of 3 comes up, throw the die again; if any other number comes up, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

***Solution:*** The sample space consists of: for each first die outcome that is a multiple of 3 (3 or 6), the second stage is another die throw (6 outcomes each); for outcomes 1,2,4,5, the second stage is a coin toss (2 outcomes each). This gives 20 equally likely outcomes in total.
Let $F$: at least one die shows a 3. This occurs either when the first throw is 3, or when the first throw is 3 or 6 and the second die throw is 3. $F = \{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(6,3)\}$, so $P(F) = \frac{7}{20}$.
Let $E$: coin shows a tail. Since every outcome in $F$ involves two die throws (no coin toss), $E \cap F = \emptyset$.

$$P(E|F) = \frac{0}{7/20} = 0$$

**Question 16**

If $P(A) = \frac{1}{2}$, $P(B) = 0$, find $P(A|B)$.

(A) 0 (B) $\frac{1}{2}$ (C) not defined (D) 1

***Solution:*** Since $P(B) = 0$, the conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$ involves division by zero, so it is **not defined**. **Correct answer: (C) not defined.**

**Question 17**

If $A$ and $B$ are events such that $P(A|B) = P(B|A)$, then:
(A) $A \subset B$ but $A \neq B$ (B) $A = B$ (C) $A \cap B = \emptyset$ (D) $P(A) = P(B)$

***Solution:***

$$P(A|B) = P(B|A) \implies \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

Assuming $P(A \cap B) \neq 0$, we can cancel it from both sides: $\frac{1}{P(B)} = \frac{1}{P(A)} \implies P(A) = P(B)$.
**Correct answer: (D) $P(A) = P(B)$.**

## 2 Exercise 13.2

**Question 1**

If $P(A) = \dfrac{3}{5}$ and $P(B) = \dfrac{1}{5}$, find $P(A \cap B)$ if $A$ and $B$ are independent events.

***Solution:*** Since $A, B$ are independent: $P(A \cap B) = P(A) \cdot P(B) = \dfrac{3}{5} \times \dfrac{1}{5} = \dfrac{3}{25}$.

**Question 2**

Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

***Solution:*** There are 26 black cards in a deck of 52. Let $A$: first card is black, $B$: second card is black.

$$P(A) = \frac{26}{52}, \qquad P(B|A) = \frac{25}{51} \text{ (without replacement)}$$

$$P(A \cap B) = P(A) \cdot P(B|A) = \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}$$

**Question 3**

A box of oranges is inspected by examining 3 randomly selected oranges drawn without replacement. If all 3 are good, the box is approved for sale, otherwise rejected. Find the probability that a box containing 15 oranges, out of which 12 are good and 3 are bad, will be approved for sale.

***Solution:*** Let $A_1, A_2, A_3$ be the events that the 1st, 2nd, 3rd orange drawn is good, respectively.

$$P(A_1) = \frac{12}{15}, \quad P(A_2|A_1) = \frac{11}{14}, \quad P(A_3|A_1 \cap A_2) = \frac{10}{13}$$

By the multiplication theorem:

$$P(A_1 \cap A_2 \cap A_3) = \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{1320}{2730} = \frac{44}{91}$$

**Question 4**

A fair coin and an unbiased die are tossed. Let $A$ be the event 'head appears on the coin' and $B$ be the event '3 on the die'. Check whether $A$ and $B$ are independent events or not.

***Solution:*** Sample space has $2 \times 6 = 12$ equally likely outcomes. $P(A) = \dfrac{6}{12} = \dfrac{1}{2}$, $P(B) = \dfrac{2}{12} = \dfrac{1}{6}$. $A \cap B = \{(H, 3)\}$, so $P(A \cap B) = \dfrac{1}{12}$. Check: $P(A) \cdot P(B) = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12} = P(A \cap B)$. Since $P(A \cap B) = P(A)P(B)$, **$A$ and $B$ are independent**.

**Question 5**

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let $A$: number obtained is even, $B$: number obtained is red. Are $A$ and $B$ independent?

***Solution:*** $A = \{2, 4, 6\}$, so $P(A) = \frac{3}{6} = \frac{1}{2}$. $B = \{1, 2, 3\}$ (red), so $P(B) = \frac{3}{6} = \frac{1}{2}$. $A \cap B = \{2\}$, so $P(A \cap B) = \frac{1}{6}$. Check: $P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$. Since $P(A \cap B) \neq P(A)P(B)$, **$A$ and $B$ are not independent**.

## Question 6

Let $E$ and $F$ be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$, $P(E \cap F) = \frac{1}{5}$. Are $E$ and $F$ independent?

***Solution:*** $P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}$. Since $P(E \cap F) = \frac{1}{5} = \frac{10}{50} \neq \frac{9}{50} = P(E)P(F)$, **$E$ and $F$ are not independent**.

## Question 7

Given that events $A$ and $B$ are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$, and $P(B) = p$. Find $p$ if they are:
(i) mutually exclusive (ii) independent

***Solution:*** **(i) Mutually exclusive:** $P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B) = 0$).

$$\frac{3}{5} = \frac{1}{2} + p \implies p = \frac{3}{5} - \frac{1}{2} = \frac{6 - 5}{10} = \frac{1}{10}$$

**(ii) Independent:** $P(A \cup B) = P(A) + P(B) - P(A)P(B)$.

$$\frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p \implies \frac{3}{5} - \frac{1}{2} = \frac{p}{2} \implies \frac{1}{10} = \frac{p}{2} \implies p = \frac{1}{5}$$

## Question 8

Let $A$ and $B$ be independent events with $P(A) = 0.3$, $P(B) = 0.4$. Find:
(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A|B)$ (iv) $P(B|A)$

***Solution:*** **(i)** $P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 0.12$
**(ii)** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.4 - 0.12 = 0.58$
**(iii)** $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3$ (equals $P(A)$, as expected for independent events)
**(iv)** $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.3} = 0.4$ (equals $P(B)$)

## Question 9

If $A$ and $B$ are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(\text{not } A \text{ and not } B)$.

***Solution:*** Check independence: $P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = P(A \cap B)$, so $A, B$ are independent, and hence $A', B'$ are also independent.

$$P(A' \cap B') = P(A') \cdot P(B') = \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{2}\right) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

## Question 10

Events $A$ and $B$ are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$. State whether $A$ and $B$ are independent.

***Solution:*** $P(\text{not } A \text{ or not } B) = P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B)$.

$$\frac{1}{4} = 1 - P(A \cap B) \implies P(A \cap B) = \frac{3}{4}$$

Check: $P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$. Since $P(A \cap B) = \frac{3}{4} = \frac{18}{24} \neq \frac{7}{24} = P(A)P(B)$, $A$ **and** $B$ **are not independent**.

## Question 11

Given two independent events $A$ and $B$ such that $P(A) = 0.3$, $P(B) = 0.6$. Find:
(i) $P(A \text{ and } B)$ (ii) $P(A \text{ and not } B)$ (iii) $P(A \text{ or } B)$ (iv) $P(\text{neither } A \text{ nor } B)$

***Solution:*** **(i)** $P(A \cap B) = 0.3 \times 0.6 = 0.18$
**(ii)** $P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.18 = 0.12$
**(iii)** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.18 = 0.72$
**(iv)** $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.72 = 0.28$

## Question 12

A die is tossed thrice. Find the probability of getting an odd number at least once.

***Solution:*** $P(\text{odd number in one toss}) = \frac{3}{6} = \frac{1}{2}$, so $P(\text{even number}) = \frac{1}{2}$.

$$P(\text{no odd number in 3 tosses}) = P(\text{even each time}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(\text{at least one odd number}) = 1 - \frac{1}{8} = \frac{7}{8}$$

## Question 13

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that:
(i) both balls are red (ii) first is black and second is red (iii) one is black and the other is red

***Solution:*** Total balls $= 18$. $P(\text{red}) = \frac{8}{18} = \frac{4}{9}$, $P(\text{black}) = \frac{10}{18} = \frac{5}{9}$. Since draws are with replacement, they are independent.
**(i)** $P(\text{both red}) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

**(ii)** $P(\text{black then red}) = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

**(iii)** $P(\text{one black, one red}) = P(\text{black then red}) + P(\text{red then black}) = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

## Question 14

Probability of solving a specific problem independently by $A$ and $B$ are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that:
(i) the problem is solved (ii) exactly one of them solves the problem

***Solution:*** $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, so $P(A') = \frac{1}{2}$, $P(B') = \frac{2}{3}$.
**(i)** $P(\text{problem solved}) = 1 - P(\text{neither solves}) = 1 - P(A')P(B') = 1 - \frac{1}{2} \times \frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}$
**(ii)** $P(\text{exactly one solves}) = P(A)P(B') + P(A')P(B) = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

## Question 15

One card is drawn at random from a well-shuffled deck of 52 cards. In which of the following cases are the events $E$ and $F$ independent?
(i) $E$: card drawn is a spade, $F$: card drawn is an ace
(ii) $E$: card drawn is black, $F$: card drawn is a king
(iii) $E$: card drawn is a king or queen, $F$: card drawn is a queen or jack

***Solution:*** **(i)** $P(E) = \frac{13}{52} = \frac{1}{4}$, $P(F) = \frac{4}{52} = \frac{1}{13}$. $E \cap F = \{\text{ace of spades}\}$, $P(E \cap F) = \frac{1}{52}$.
$P(E)P(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} = P(E \cap F)$. **Independent.**
**(ii)** $P(E) = \frac{26}{52} = \frac{1}{2}$, $P(F) = \frac{4}{52} = \frac{1}{13}$. $E \cap F = \{\text{black kings}\} = 2$ cards, $P(E \cap F) = \frac{2}{52} = \frac{1}{26}$.
$P(E)P(F) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} = P(E \cap F)$. **Independent.**
**(iii)** $P(E) = \frac{8}{52} = \frac{2}{13}$, $P(F) = \frac{8}{52} = \frac{2}{13}$. $E \cap F = \{\text{queens}\} = 4$ cards, $P(E \cap F) = \frac{4}{52} = \frac{1}{13}$.
$P(E)P(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169} \neq \frac{1}{13} = P(E \cap F)$. **Not independent.**

## Question 16

In a hostel, 60% of students read Hindi newspaper, 40% read English newspaper, and 20% read both. A student is selected at random.
(a) Find the probability that she reads neither Hindi nor English newspaper.
(b) If she reads Hindi, find the probability she reads English too.
(c) If she reads English, find the probability she reads Hindi too.

***Solution:*** Let $H$: reads Hindi, $E$: reads English. $P(H) = 0.6$, $P(E) = 0.4$, $P(H \cap E) = 0.2$.
**(a)** $P(H' \cap E') = 1 - P(H \cup E) = 1 - [P(H) + P(E) - P(H \cap E)] = 1 - [0.6 + 0.4 - 0.2] = 1 - 0.8 = 0.2$
**(b)** $P(E|H) = \frac{P(H \cap E)}{P(H)} = \frac{0.2}{0.6} = \frac{1}{3}$
**(c)** $P(H|E) = \frac{P(H \cap E)}{P(E)} = \frac{0.2}{0.4} = \frac{1}{2}$

## Question 17

The probability of obtaining an even prime number on each die when a pair of dice is rolled is:
(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{12}$ (D) $\frac{1}{36}$

***Solution:*** The only even prime number is 2. Probability of getting 2 on one die $= \frac{1}{6}$. Since the two dice are independent: $P(2 \text{ on both dice}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. **Correct answer: (D) $\frac{1}{36}$.**

## Question 18

Two events $A$ and $B$ will be independent if:
(A) $A$ and $B$ are mutually exclusive
(B) $P(A'B') = [1 - P(A)][1 - P(B)]$
(C) $P(A) = P(B)$
(D) $P(A) + P(B) = 1$

***Solution:*** For independent events, $P(A \cap B) = P(A)P(B)$, and it follows that $P(A' \cap B') = P(A')P(B') = [1 - P(A)][1 - P(B)]$; this equivalence characterizes independence. Option (A) describes mutually exclusive events, which is a different (and generally incompatible) condition. Options (C) and (D) are unrelated to independence. **Correct answer: (B) $P(A'B') = [1 - P(A)][1 - P(B)]$.**

## 3 Exercise 13.3

### Question 1

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted, and it is put back in the urn. Moreover, 2 additional balls of the colour drawn are put in the urn, and then a ball is drawn at random. What is the probability that the second ball is red?

***Solution:*** Let $E_1$: first ball drawn is red, $E_2$: first ball drawn is black. $P(E_1) = P(E_2) = \frac{1}{2}$. Let $A$: second ball drawn is red.
If first ball is red (2 more red balls added): urn has 7 red, 5 black (12 total). $P(A|E_1) = \frac{7}{12}$. If first ball is black (2 more black balls added): urn has 5 red, 7 black (12 total). $P(A|E_2) = \frac{5}{12}$.
By the theorem of total probability:

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) = \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{7}{24} + \frac{5}{24} = \frac{12}{24} = \frac{1}{2}$$

### Question 2

A bag contains 4 red and 4 black balls; another bag contains 2 red and 6 black balls. One of the two bags is selected at random, and a ball is drawn from it, which is found to be red. Find the probability that the ball is drawn from the first bag.

***Solution:*** Let $E_1$: bag 1 selected, $E_2$: bag 2 selected. $P(E_1) = P(E_2) = \frac{1}{2}$. Let $A$: ball drawn is red. $P(A|E_1) = \frac{4}{8} = \frac{1}{2}$, $P(A|E_2) = \frac{2}{8} = \frac{1}{4}$.
By Bayes' theorem:

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}} = \frac{1/4}{1/4 + 1/8} = \frac{1/4}{3/8} = \frac{2}{3}$$

### Question 3

Of the students in a college, 60% reside in the hostel and 40% are day scholars. Previous results show that 30% of hostelers attain A grade, while 20% of day scholars attain A grade. At the end of the year, a student is chosen at random and has an A grade. What is the probability that the student is a hosteler?

***Solution:*** Let $E_1$: hosteler, $E_2$: day scholar. $P(E_1) = 0.6$, $P(E_2) = 0.4$. Let $A$: student gets A grade. $P(A|E_1) = 0.3$, $P(A|E_2) = 0.2$.
By Bayes' theorem:

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2} = \frac{0.18}{0.18 + 0.08} = \frac{0.18}{0.26} = \frac{9}{13}$$

### Question 4

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses.

Assuming that a student who guesses will be correct with probability $\frac{1}{4}$, what is the probability that the student knows the answer, given that he answered it correctly?

***Solution:*** Let $E_1$: student knows the answer, $E_2$: student guesses. $P(E_1) = \frac{3}{4}$, $P(E_2) = \frac{1}{4}$. Let $A$: answer is correct. $P(A|E_1) = 1$ (if he knows, he's certainly correct), $P(A|E_2) = \frac{1}{4}$.
By Bayes' theorem:

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}} = \frac{3/4}{3/4 + 1/16} = \frac{3/4}{13/16} = \frac{12}{13}$$

## Question 5

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact present. However, the test also yields a false positive result for 0.5% of healthy persons tested. If 0.1% of the population actually has the disease, what is the probability that a person has the disease, given that the test result is positive?

***Solution:*** Let $E_1$: person has the disease, $E_2$: person does not have the disease. $P(E_1) = 0.001$, $P(E_2) = 0.999$. Let $A$: test result is positive. $P(A|E_1) = 0.99$, $P(A|E_2) = 0.005$.
By Bayes' theorem:

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995} = \frac{0.00099}{0.005985} = \frac{99}{599.5} \approx \frac{22}{133} \approx 0.1652$$

More precisely, $P(E_1|A) = \frac{990}{5985} = \frac{22}{133}$.

## Question 6

There are three coins: one is a two-headed coin, another is a biased coin that comes up heads 75% of the time, and the third is an unbiased coin. One of the three coins is chosen at random and tossed; it shows heads. What is the probability that it was the two-headed coin?

***Solution:*** Let $E_1$: two-headed coin, $E_2$: biased coin, $E_3$: unbiased coin. $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$. Let $A$: coin shows heads. $P(A|E_1) = 1$, $P(A|E_2) = 0.75 = \frac{3}{4}$, $P(A|E_3) = \frac{1}{2}$.
By Bayes' theorem:

$$P(E_1|A) = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{1}{1 + \frac{3}{4} + \frac{1}{2}} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$$

## Question 7

An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probabilities of an accident are 0.01, 0.03, and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

***Solution:*** Total drivers $= 2000 + 4000 + 6000 = 12000$. $P(E_1) = \frac{2000}{12000} = \frac{1}{6}$ (scooter), $P(E_2) = \frac{4000}{12000} = \frac{1}{3}$ (car), $P(E_3) = \frac{6000}{12000} = \frac{1}{2}$ (truck). $P(A|E_1) = 0.01$, $P(A|E_2) = 0.03$, $P(A|E_3) = 0.15$.
By Bayes' theorem:

$$P(E_1|A) = \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15} = \frac{0.001667}{0.001667 + 0.01 + 0.075} = \frac{0.001667}{0.086667} = \frac{1}{52}$$

## Question 8

A factory has two machines A and B. Past record shows that machine A produced 60% of items and machine B produced 40% of items. Further, 2% of items produced by machine A and 1% by machine B were defective. All items are put into one stockpile, and then one item is chosen at random and found to be defective. What is the probability that it was produced by machine B?

***Solution:*** Let $E_1$: item from machine A, $E_2$: item from machine B. $P(E_1) = 0.6$, $P(E_2) = 0.4$. Let $D$: item is defective. $P(D|E_1) = 0.02$, $P(D|E_2) = 0.01$.
By Bayes' theorem:

$$P(E_2|D) = \frac{P(E_2)P(D|E_2)}{P(E_1)P(D|E_1) + P(E_2)P(D|E_2)} = \frac{0.4 \times 0.01}{0.6 \times 0.02 + 0.4 \times 0.01} = \frac{0.004}{0.012 + 0.004} = \frac{0.004}{0.016} = \frac{1}{4}$$

## Question 9

Two groups are competing for the position on the board of directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. If the first group wins, the probability of introducing a new product is 0.7, and the corresponding probability if the second group wins is 0.3. Find the probability that the new product introduced was by the second group.

***Solution:*** Let $E_1$: first group wins, $E_2$: second group wins. $P(E_1) = 0.6$, $P(E_2) = 0.4$. Let $A$: new product introduced. $P(A|E_1) = 0.7$, $P(A|E_2) = 0.3$.
By Bayes' theorem:

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3} = \frac{0.12}{0.42 + 0.12} = \frac{0.12}{0.54} = \frac{2}{9}$$

## Question 10

Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3, or 4, she tosses a coin once and notes if it is head or tail. If she obtained exactly one head, what is the probability that she threw 1, 2, 3, or 4 with the die?

***Solution:*** Let $E_1$: she gets 5 or 6, $E_2$: she gets 1, 2, 3, or 4. $P(E_1) = \frac{2}{6} = \frac{1}{3}$, $P(E_2) = \frac{4}{6} = \frac{2}{3}$.
Let $A$: exactly one head is obtained.

If $E_1$ occurs, coin is tossed 3 times: $P(A|E_1) = P(\text{exactly one head in 3 tosses}) = \binom{3}{1}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$.

If $E_2$ occurs, coin is tossed once: $P(A|E_2) = P(\text{head}) = \frac{1}{2}$.

By Bayes' theorem:

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{1/3}{1/8 + 1/3}$$

$$= \frac{1/3}{3/24 + 8/24} = \frac{1/3}{11/24} = \frac{1}{3} \times \frac{24}{11} = \frac{8}{11}$$

## Question 11

A manufacturer has three machine operators A, B, and C. The first operator A produces 1% defective items, B produces 5% defective items, and C produces 7% defective items. A is on the job 50% of the time, B is on the job 30% of the time, and C is on the job 20% of the time. A defective item is produced. What is the probability that it was produced by A?

***Solution:*** Let $E_1, E_2, E_3$ be the events that the item is produced by A, B, C respectively. $P(E_1) = 0.5$, $P(E_2) = 0.3$, $P(E_3) = 0.2$. Let $D$: item is defective. $P(D|E_1) = 0.01$, $P(D|E_2) = 0.05$, $P(D|E_3) = 0.07$.
By Bayes' theorem:

$$P(E_1|D) = \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07} = \frac{0.005}{0.005 + 0.015 + 0.014} = \frac{0.005}{0.034} = \frac{5}{34}$$

## Question 12

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

***Solution:*** Let $E_1$: lost card is a diamond, $E_2$: lost card is not a diamond. $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{39}{52} = \frac{3}{4}$. Let $A$: two cards drawn from the remaining 51 are both diamonds.

If lost card is diamond, 12 diamonds remain among 51 cards: $P(A|E_1) = \frac{\binom{12}{2}}{\binom{51}{2}} = \frac{66}{1275}$.

If lost card is not diamond, 13 diamonds remain among 51 cards: $P(A|E_2) = \frac{\binom{13}{2}}{\binom{51}{2}} = \frac{78}{1275}$.

By Bayes' theorem:

$$P(E_1|A) = \frac{\frac{1}{4} \times \frac{66}{1275}}{\frac{1}{4} \times \frac{66}{1275} + \frac{3}{4} \times \frac{78}{1275}} = \frac{\frac{66}{4}}{\frac{66}{4} + \frac{234}{4}} = \frac{66}{66 + 234} = \frac{66}{300} = \frac{11}{50}$$

## Question 13

The probability that A speaks the truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. Find the probability that actually there was a head.

***Solution:*** Let $E_1$: head actually appears, $E_2$: tail actually appears. $P(E_1) = P(E_2) = \frac{1}{2}$. Let $A$: A reports a head. $P(A|E_1) = \frac{4}{5}$ (A speaks truth), $P(A|E_2) = \frac{1}{5}$ (A lies).

By Bayes' theorem:

$$P(E_1|A) = \frac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{1}{2} \times \frac{1}{5}} = \frac{4/10}{4/10 + 1/10} = \frac{4}{5}$$

## Question 14

If $A$ and $B$ are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(A) $P(A|B) = \dfrac{P(B)}{P(A)}$ (B) $P(A|B) < P(A)$ (C) $P(A|B) \geq P(A)$ (D) None of these

***Solution:*** Since $A \subset B$, we have $A \cap B = A$, so $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$. Since $A \subset B$, $P(A) \leq P(B)$, so $\dfrac{P(A)}{P(B)} \geq P(A)$ whenever $P(B) \leq 1$ (always true), meaning $P(A|B) \geq P(A)$.

**Correct answer: (C)** $P(A|B) \geq P(A)$**.**

## 4 Miscellaneous Exercise

**Question 1**

$A$ and $B$ are two events such that $P(A) \neq 0$. Find $P(B|A)$, if:
(i) $A$ is a subset of $B$ (ii) $A \cap B = \emptyset$

***Solution:*** **(i)** If $A \subset B$, then $A \cap B = A$, so

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

**(ii)** If $A \cap B = \emptyset$, then $P(A \cap B) = 0$, so

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{P(A)} = 0$$

**Question 2**

A family has three children. Find the probability that all are boys, given that at least one of them is a boy.

***Solution:*** Sample space for 3 children (each equally likely to be a boy B or girl G) has $2^3 = 8$ outcomes:

$$S = \{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\}$$

Let $F$: at least one boy. This excludes only $GGG$, so $P(F) = \frac{7}{8}$ (7 outcomes). Let $E$: all are boys $= \{BBB\}$. Since $BBB$ has at least one boy, $E \cap F = \{BBB\}$, so $P(E \cap F) = \frac{1}{8}$.

$$P(E|F) = \frac{1/8}{7/8} = \frac{1}{7}$$

**Question 3**

Suppose that 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. Find the probability of this person being male, assuming there are an equal number of males and females.

***Solution:*** Let $E_1$: person is male, $E_2$: person is female. $P(E_1) = P(E_2) = \frac{1}{2}$. Let $A$: person has grey hair. $P(A|E_1) = 0.05$, $P(A|E_2) = 0.0025$.
By Bayes' theorem:

$$P(E_1|A) = \frac{\frac{1}{2} \times 0.05}{\frac{1}{2} \times 0.05 + \frac{1}{2} \times 0.0025} = \frac{0.05}{0.05 + 0.0025} = \frac{0.05}{0.0525} = \frac{500}{525} = \frac{20}{21}$$

**Question 4**

Prove that if $A$ and $B$ are independent events, then the events $A'$ and $B'$ are also independent.

***Solution:*** Given: $A, B$ independent, so $P(A \cap B) = P(A)P(B)$.

$$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - P(A) - P(B) + P(A)P(B) = [1 - P(A)][1 - P(B)] = P(A') \cdot P(B')$$

Since $P(A' \cap B') = P(A')P(B')$, the events $A'$ and $B'$ are independent. ■

## Question 5

A speaks the truth in 75% of cases, and B speaks the truth in 80% of cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

***Solution:*** Let $T_A$: A speaks the truth, $T_B$: B speaks the truth. $P(T_A) = 0.75$, $P(T_B) = 0.80$, so $P(T'_A) = 0.25$, $P(T'_B) = 0.20$.
They contradict each other when one speaks the truth and the other lies:

$$P(\text{contradict}) = P(T_A)P(T'_B) + P(T'_A)P(T_B) = 0.75 \times 0.20 + 0.25 \times 0.80$$

$$= 0.15 + 0.20 = 0.35$$

So they contradict each other in **35%** of the cases.

## Question 6

A die is thrown twice, and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

***Solution:*** Let $F$: sum is 6, so $F = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$, $P(F) = \frac{5}{36}$. Let $E$: 4 appears at least once. $E \cap F = \{(2,4), (4,2)\}$, so $P(E \cap F) = \frac{2}{36}$.

$$P(E|F) = \frac{2/36}{5/36} = \frac{2}{5}$$

## Question 7

Three coins are tossed simultaneously. Consider the event $E$: 'three heads or three tails', $F$: 'at least two heads', and $G$: 'at most two heads'. Determine whether the pairs of events $(E, F)$ and $(E, G)$ are independent.

***Solution:*** Sample space $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$, 8 equally likely outcomes. $E = \{HHH, TTT\}$, $P(E) = \frac{2}{8} = \frac{1}{4}$. $F = \{HHH, HHT, HTH, THH\}$, $P(F) = \frac{4}{8} = \frac{1}{2}$. $G = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$, $P(G) = \frac{7}{8}$.
**Pair $(E, F)$:** $E \cap F = \{HHH\}$, $P(E \cap F) = \frac{1}{8}$. $P(E)P(F) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = P(E \cap F)$. **$E$ and $F$ are independent.**
**Pair $(E, G)$:** $E \cap G = \{TTT\}$, $P(E \cap G) = \frac{1}{8}$. $P(E)P(G) = \frac{1}{4} \times \frac{7}{8} = \frac{7}{32} \neq \frac{1}{8} = P(E \cap G)$. **$E$ and $G$ are not independent.**

### Question 8

Probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs, none will fuse after 150 days, given that the bulbs function independently.

***Solution:*** $P(\text{bulb fuses}) = 0.05$, so $P(\text{bulb does not fuse}) = 0.95$. Since the bulbs function independently:

$$P(\text{none of the 5 fuses}) = (0.95)^5 \approx 0.7738$$

### Question 9

Suppose 15 men out of 300 and 25 women out of 1000 are good orators. Find the probability that a good orator, chosen at random, is a male, assuming males and females are in equal number.

***Solution:*** Let $E_1$: person is male, $E_2$: person is female. $P(E_1) = P(E_2) = \dfrac{1}{2}$. Let $A$: person is a good orator. $P(A|E_1) = \dfrac{15}{300} = \dfrac{1}{20}$, $P(A|E_2) = \dfrac{25}{1000} = \dfrac{1}{40}$.
By Bayes' theorem:

$$P(E_1|A) = \frac{\frac{1}{2} \times \frac{1}{20}}{\frac{1}{2} \times \frac{1}{20} + \frac{1}{2} \times \frac{1}{40}} = \frac{1/40}{1/40 + 1/80} = \frac{1/40}{3/80} = \frac{2}{3}$$

### Question 10

An urn contains $m$ white and $n$ black balls. A ball is drawn at random and put back into the urn along with $k$ additional balls of the same colour as the one drawn. A ball is again drawn at random. Show that the probability of the second ball being white does not depend on $k$.

***Solution:*** Let $E_1$: first ball drawn is white, $E_2$: first ball drawn is black. $P(E_1) = \dfrac{m}{m+n}$, $P(E_2) = \dfrac{n}{m+n}$. Let $A$: second ball drawn is white.
If $E_1$ occurs, $k$ additional white balls are added: urn has $(m+k)$ white and $n$ black, total $(m+n+k)$.

$$P(A|E_1) = \frac{m+k}{m+n+k}$$

If $E_2$ occurs, $k$ additional black balls are added: urn has $m$ white and $(n+k)$ black, total $(m+n+k)$.

$$P(A|E_2) = \frac{m}{m+n+k}$$

By the theorem of total probability:

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) = \frac{m}{m+n} \cdot \frac{m+k}{m+n+k} + \frac{n}{m+n} \cdot \frac{m}{m+n+k}$$

$$= \frac{m(m+k) + nm}{(m+n)(m+n+k)} = \frac{m(m+k+n)}{(m+n)(m+n+k)} = \frac{m}{m+n}$$

Since this simplifies to $\dfrac{m}{m+n}$, which is independent of $k$, the probability that the second ball is white **does not depend on** $k$. ■

### Question 11

If $A$ and $B$ are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then:
(A) $A \subset B$ (B) $B \subset A$ (C) $B = \emptyset$ (D) $A = \emptyset$

***Solution:*** $P(B|A) = 1$ means $\frac{P(A \cap B)}{P(A)} = 1$, so $P(A \cap B) = P(A)$. Since $A \cap B \subseteq A$ and $P(A \cap B) = P(A)$, this means $A \cap B = A$ (in terms of measure), i.e., $A \subset B$. **Correct answer: (A) $A \subset B$.**

### Question 12

If $P(A|B) > P(A)$, then which of the following is correct?
(A) $P(B|A) < P(B)$ (B) $P(A \cap B) < P(A) \cdot P(B)$
(C) $P(B|A) > P(B)$ (D) $P(B|A) = P(B)$

***Solution:*** Given $P(A|B) > P(A)$: $\frac{P(A \cap B)}{P(B)} > P(A) \implies P(A \cap B) > P(A)P(B)$. Dividing both sides by $P(A)$: $\frac{P(A \cap B)}{P(A)} > P(B) \implies P(B|A) > P(B)$. **Correct answer: (C) $P(B|A) > P(B)$.**

### Question 13

If $A$ and $B$ are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then:
(A) $P(B|A) = 1$ (B) $P(A|B) = 1$ (C) $P(B|A) = 0$ (D) $P(A|B) = 0$

***Solution:***

$$P(A) + P(B) - P(A \cap B) = P(A) \implies P(B) - P(A \cap B) = 0 \implies P(A \cap B) = P(B)$$

Then $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$. **Correct answer: (B) $P(A|B) = 1$.**

## End of Chapter 13: Probability

All questions from Exercises 13.1, 13.2, 13.3 and the Miscellaneous Exercise are covered above, matching the rationalised CBSE 2026-27 syllabus.

*Chapter-wise NCERT Maths Solutions – Class 12*