NCERT Class 12 Maths

Chapter-wise Solutions

Chapter 7

Integrals

Complete Step-by-Step NCERT Solutions

In this document

Remaining exercises (7.2 – 7.11 and Miscellaneous) to follow in continuation parts of this solution series.

For Class 12 CBSE / NCERT students

Every question solved with clear, exam-ready working

Prepared as a print-friendly, exam-preparation resource

Chapter 7: Integrals — Overview

Integration is the **inverse process of differentiation**. If $\frac{d}{dx}F(x) = f(x)$, then $F(x)$ is called an **antiderivative** (or indefinite integral) of $f(x)$, written as

$$\int f(x)\,dx = F(x) + C$$

where C is the **constant of integration**. Since the derivative of a constant is zero, a family of curves (differing only by C) all have the same derivative $f(x)$ – hence "indefinite" integral.

Key Formulas Used

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ (n \neq -1)$

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

Linearity: $\int [af(x) + bg(x)]\,dx$
$= a\int f(x)\,dx + b\int g(x)\,dx$

Chain rule form: $\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + C$

These standard results are exactly what we use, in combination with algebraic simplification, to solve every problem in Exercise 7.1 below.

1 Exercise 7.1

Find an antiderivative of the following functions by the method of inspection (Q1–5), and evaluate the following integrals (Q6–20). Choose the correct answer in Q21–22.

Questions 1 – 5: Method of Inspection

Question 1. $\sin 2x$
Solution:
We need a function whose derivative is $\sin 2x$. Since $\frac{d}{dx}(\cos 2x) = -2\sin 2x$,

$$\sin 2x = -\frac{1}{2}\cdot\frac{d}{dx}(\cos 2x) = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Answer: $-\frac{1}{2}\cos 2x + C$

Question 2. $\cos 3x$
Solution:
Since $\frac{d}{dx}(\sin 3x) = 3\cos 3x$,

$$\cos 3x = \frac{1}{3}\cdot\frac{d}{dx}(\sin 3x) = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Answer: $\frac{1}{3}\sin 3x + C$

Question 3. e^{2x}

Solution:

Since $\frac{d}{dx}(e^{2x}) = 2e^{2x}$,

$$e^{2x} = \frac{1}{2} \cdot \frac{d}{dx}(e^{2x}) = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Answer: $\frac{1}{2}e^{2x} + C$

Question 4. $(ax+b)^2$

Solution:

Since $\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$,

$$(ax+b)^2 = \frac{1}{3a} \cdot \frac{d}{dx}(ax+b)^3 = \frac{d}{dx}\left(\frac{(ax+b)^3}{3a}\right)$$

Answer: $\frac{(ax+b)^3}{3a} + C$

Question 5. $\sin 2x - 4e^{3x}$

Solution:

Using results of Q1 and the method of Q3, along with linearity of the antiderivative,

$$\int (\sin 2x - 4e^{3x})\,dx = \int \sin 2x\,dx - 4\int e^{3x}\,dx = -\frac{1}{2}\cos 2x - 4 \cdot \frac{1}{3}e^{3x}$$

Answer: $-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x} + C$

Questions 6 – 11

Question 6. $\int (4e^{3x} + 1)\,dx$

Solution:

$$\int (4e^{3x}+1)\,dx = 4\int e^{3x}dx + \int 1\,dx = 4 \cdot \frac{e^{3x}}{3} + x$$

Answer: $\frac{4}{3}e^{3x} + x + C$

Question 7. $\int x^2\left(1 - \frac{1}{x^2}\right)dx$

Solution:

Expand the integrand first:

$$\int x^2\left(1 - \frac{1}{x^2}\right)dx = \int (x^2 - 1)\,dx = \frac{x^3}{3} - x$$

Answer: $\frac{x^3}{3} - x + C$

Question 8. $\int (ax^2 + bx + c)\,dx$

Solution:
Integrate term by term using $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$\int (ax^2 + bx + c)\,dx = a \cdot \frac{x^3}{3} + b \cdot \frac{x^2}{2} + cx$$

Answer: $\dfrac{ax^3}{3} + \dfrac{bx^2}{2} + cx + C$

Question 9. $\int (2x^2 + e^x)\,dx$

Solution:

$$\int (2x^2 + e^x)\,dx = 2 \cdot \frac{x^3}{3} + e^x$$

Answer: $\dfrac{2}{3}x^3 + e^x + C$

Question 10. $\int \left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2 dx$

Solution:
Expand using $(A - B)^2 = A^2 - 2AB + B^2$:

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x - 2 + \frac{1}{x}$$

$$\int \left(x - 2 + \frac{1}{x}\right) dx = \frac{x^2}{2} - 2x + \ln|x|$$

Answer: $\dfrac{x^2}{2} - 2x + \ln|x| + C$

Question 11. $\int \dfrac{x^3 + 5x^2 - 4}{x^2}\,dx$

Solution:
Split the fraction term by term:

$$\frac{x^3 + 5x^2 - 4}{x^2} = x + 5 - \frac{4}{x^2}$$

$$\int \left(x + 5 - \frac{4}{x^2}\right) dx = \frac{x^2}{2} + 5x - 4 \cdot \frac{x^{-1}}{-1} = \frac{x^2}{2} + 5x + \frac{4}{x}$$

Answer: $\dfrac{x^2}{2} + 5x + \dfrac{4}{x} + C$

Questions 12 – 16

Question 12. $\int \dfrac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

Solution:

Divide each term by $\sqrt{x} = x^{1/2}$:

$$\frac{x^3 + 3x + 4}{\sqrt{x}} = x^{5/2} + 3x^{1/2} + 4x^{-1/2}$$

$$\int \left(x^{5/2} + 3x^{1/2} + 4x^{-1/2}\right) dx = \frac{x^{7/2}}{7/2} + 3 \cdot \frac{x^{3/2}}{3/2} + 4 \cdot \frac{x^{1/2}}{1/2}$$

$$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8x^{1/2}$$

Answer: $\frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

Question 13. $\displaystyle\int \frac{x^3 - x^2 + x - 1}{x - 1}\, dx$

Solution:

Factor the numerator by grouping:

$$x^3 - x^2 + x - 1 = x^2(x-1) + 1(x-1) = (x-1)(x^2+1)$$

So for $x \neq 1$, the integrand simplifies:

$$\frac{(x-1)(x^2+1)}{x-1} = x^2 + 1$$

$$\int (x^2 + 1)\, dx = \frac{x^3}{3} + x$$

Answer: $\frac{x^3}{3} + x + C$

Question 14. $\displaystyle\int (1-x)\sqrt{x}\, dx$

Solution:

Expand:

$$(1-x)\sqrt{x} = x^{1/2} - x^{3/2}$$

$$\int \left(x^{1/2} - x^{3/2}\right) dx = \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} = \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}$$

Answer: $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$

Question 15. $\displaystyle\int \sqrt{x}\,(3x^2 + 2x + 3)\, dx$

Solution:

Multiply through by $x^{1/2}$:

$$\sqrt{x}(3x^2 + 2x + 3) = 3x^{5/2} + 2x^{3/2} + 3x^{1/2}$$

$$\int \left(3x^{5/2} + 2x^{3/2} + 3x^{1/2}\right) dx = 3 \cdot \frac{2}{7}x^{7/2} + 2 \cdot \frac{2}{5}x^{5/2} + 3 \cdot \frac{2}{3}x^{3/2}$$

$$= \frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2}$$

Answer: $\frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + C$

Question 16. $\int (2x - 3\cos x + e^x)\, dx$

Solution:

$$\int (2x - 3\cos x + e^x)\, dx = x^2 - 3\sin x + e^x$$

Answer: $x^2 - 3\sin x + e^x + C$

Questions 17 – 20

Question 17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

Solution:

$$\int \left(2x^2 - 3\sin x + 5x^{1/2}\right) dx = 2\cdot\frac{x^3}{3} - 3(-\cos x) + 5\cdot\frac{2}{3}x^{3/2}$$

Answer: $\frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + C$

Question 18. $\int \sec x(\sec x + \tan x)\, dx$

Solution:
Expand the product:

$$\sec x(\sec x + \tan x) = \sec^2 x + \sec x \tan x$$

Using $\int \sec^2 x\, dx = \tan x$ and $\int \sec x \tan x\, dx = \sec x$:

Answer: $\tan x + \sec x + C$

Question 19. $\int \frac{\sec^2 x}{\csc^2 x}\, dx$

Solution:
Rewrite in terms of sine and cosine:

$$\frac{\sec^2 x}{\csc^2 x} = \frac{1/\cos^2 x}{1/\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

Using the identity $\tan^2 x = \sec^2 x - 1$:

$$\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx = \tan x - x$$

Answer: $\tan x - x + C$

Question 20. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx$

Solution:
Split the fraction:

$$\frac{2 - 3\sin x}{\cos^2 x} = \frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} = 2\sec^2 x - 3\sec x \tan x$$

$$\int (2\sec^2 x - 3\sec x \tan x)\, dx = 2\tan x - 3\sec x$$

Answer: $2\tan x - 3\sec x + C$

Questions 21 – 22: Multiple Choice Questions

Question 21. The antiderivative of $\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$ equals:

(A) $\dfrac{1}{3}x^{1/3} + 2x^{1/2} + C$ (B) $\dfrac{2}{3}x^{2/3} + \dfrac{1}{2}x^2 + C$

(C) $\dfrac{2}{3}x^{3/2} + 2x^{1/2} + C$ (D) $\dfrac{3}{2}x^{3/2} + \dfrac{1}{2}x^{1/2} + C$

Solution:

$$\int \left(x^{1/2} + x^{-1/2}\right) dx = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

This matches option (C).

Answer: Option (C) is correct.

Question 22. If $\dfrac{d}{dx}f(x) = 4x^3 - \dfrac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is:

(A) $x^4 + \dfrac{1}{x^3} - \dfrac{129}{8}$ (B) $x^3 + \dfrac{1}{x^4} + \dfrac{129}{8}$

(C) $x^4 + \dfrac{1}{x^3} + \dfrac{129}{8}$ (D) $x^3 + \dfrac{1}{x^4} - \dfrac{129}{8}$

Solution:

Integrate the derivative to recover $f(x)$:

$$f(x) = \int \left(4x^3 - 3x^{-4}\right) dx = 4 \cdot \frac{x^4}{4} - 3 \cdot \frac{x^{-3}}{-3} = x^4 + \frac{1}{x^3} + C \quad \text{...(i)}$$

Apply the condition $f(2) = 0$:

$$0 = (2)^4 + \frac{1}{(2)^3} + C = 16 + \frac{1}{8} + C \;\Rightarrow\; C = -16 - \frac{1}{8} = -\frac{129}{8}$$

Substituting back into (i):

$$f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

This matches option (A).

Answer: Option (A) is correct.

End of Exercise 7.1

All 22 questions solved. Next in this series: **Exercise 7.2** (Integration by Substitution) and onward, continuing the complete Chapter 7 solution set.