NCERT Solutions

Class 12 Mathematics

Chapter 1: Relations and Functions

Exercise 1.1 – Full Step-by-Step Solutions (Questions 1–16)

This document is part of a chapter-wise NCERT Class 12 Maths solutions series. Each exercise question is solved with complete, step-by-step working.

Chapter 1: Relations and Functions

Exercise 1.1

Question 1

Determine whether each of the following relations are reflexive, symmetric and transitive:

Solution:

(i) Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y): $3x - y = 0$}

Since $3x - y = 0 \Rightarrow y = 3x$, the pairs in R are (1,3), (2,6), (3,9), (4,12).

Reflexive: (1,1) needs y=3x i.e. 1=3, which is false. So $(1,1) \notin R \Rightarrow$ R is **not reflexive**.

Symmetric: $(1,3) \in R$, but (3,1) needs $3(3)-1=8\neq0$, so $(3,1) \notin R \Rightarrow$ **not symmetric**.

Transitive: $(1,3) \in R$ and $(3,9) \in R$, but (1,9) needs $3(1)-9=-6\neq0$, so $(1,9)\notin R \Rightarrow$ **not transitive**.

Answer: R is neither reflexive, nor symmetric, nor transitive.

Solution:

(ii) Relation R in N defined as R = {(x, y): $y = x + 5$ and $x < 4$}

x can only be 1, 2, 3 (natural numbers less than 4), giving pairs (1,6), (2,7), (3,8).

Reflexive: (1,1) would need 1=1+5=6, false $\Rightarrow$ **not reflexive**.

Symmetric: $(1,6)\in R$ but (6,1) needs 1=6+5=11, false $\Rightarrow$ **not symmetric**.

Transitive: The second element of every pair (6,7,8) never appears as a first element of any pair, so there is no chain (a,b),(b,c) to check $\Rightarrow$ the condition holds **vacuously true**, so R **is transitive**.

Answer: R is neither reflexive nor symmetric, but it is transitive.

Solution:

(iii) Relation R in A = {1,2,3,4,5,6} as R = {(x,y): y is divisible by x}

Reflexive: Every x divides itself, so $(x,x) \in R$ for all x $\Rightarrow$ **reflexive**.

Symmetric: $(2,4) \in R$ since 4 is divisible by 2, but (4,2) needs 2 divisible by 4, false $\Rightarrow$ **not symmetric**.

Transitive: If x divides y and y divides z, then x divides z (property of divisibility) $\Rightarrow$ **transitive**.

Answer: R is reflexive and transitive, but not symmetric.

Solution:

(iv) Relation R in Z (integers) defined as R = {(x,y): $x - y$ is an integer}

Since x and y are both integers, $x - y$ is always an integer, so R contains every pair of integers.

Reflexive: $x - x = 0$, an integer $\Rightarrow$ **reflexive**.

Symmetric: If $x-y$ is an integer, then $y-x = -(x-y)$ is also an integer $\Rightarrow$ **symmetric**.

Transitive: If $x-y$ and $y-z$ are integers, their sum $x-z$ is an integer $\Rightarrow$ **transitive**.

Answer: R is reflexive, symmetric and transitive — hence R is an equivalence relation.

Solution:

(v) Relation R in the set A of human beings in a town at a particular time:

(a) R = {(x,y): x and y work at the same place}

Reflexive (x works where x works), symmetric (if x,y share workplace, so do y,x), transitive (if x,y share workplace and y,z share workplace, so do x,z).

Answer: R is reflexive, symmetric and transitive — an equivalence relation.

Solution:

(b) R = {(x,y): x and y live in the same locality} — by the identical reasoning as (a):

Answer: R is reflexive, symmetric and transitive — an equivalence relation.

Solution:

(c) R = {(x,y): x is exactly 7 cm taller than y}

Reflexive: x cannot be 7 cm taller than itself $\Rightarrow$ **not reflexive**.

Symmetric: If x is 7 cm taller than y, y is 7 cm shorter (not taller) than x $\Rightarrow$ **not symmetric**.

Transitive: If x is 7 cm taller than y, and y is 7 cm taller than z, then x is 14 cm (not 7 cm) taller than z $\Rightarrow$ **not transitive**.

Answer: R is neither reflexive, nor symmetric, nor transitive.

Solution:

(d) R = {(x,y): x is wife of y}

Reflexive: x cannot be wife of herself $\Rightarrow$ **not reflexive**.

Symmetric: If x is wife of y, y (husband) is not wife of x $\Rightarrow$ **not symmetric**.

Transitive: If $(x,y) \in R$, y is male, so y cannot also be someone's wife; there is no pair $(y,z) \in R$ to test, so the condition holds vacuously $\Rightarrow$ **transitive**.

Answer: R is neither reflexive nor symmetric, but it is transitive.

Solution:

(e) R = {(x,y): x is father of y}

Reflexive: x cannot be father of himself $\Rightarrow$ **not reflexive**.

Symmetric: If x is father of y, y is not father of x $\Rightarrow$ **not symmetric**.

Transitive: If x is father of y and y is father of z, x is grandfather (not father) of z $\Rightarrow$ **not transitive**.

Answer: R is neither reflexive, nor symmetric, nor transitive.

Question 2

Show that the relation R in the set R of real numbers, defined as $R = \{(a,b): a \leq b^2\}$, is neither reflexive, nor symmetric, nor transitive.

Solution:

Reflexive: Take $a = 1/2$. Then $a \leq a^2$ means $1/2 \leq 1/4$, which is false. So $(1/2, 1/2) \notin R \Rightarrow$ **not reflexive**.

Symmetric: Take $a = -1$, $b = 2$. Then $a \leq b^2 \Rightarrow -1 \leq 4$, true, so $(-1,2) \in R$. But $b \leq a^2 \Rightarrow 2 \leq 1$, false, so $(2,-1) \notin R \Rightarrow$ **not symmetric**.

Transitive: Take $a = 3$, $b = -2$, $c = 1$. Check $(3,-2)$: $3 \leq (-2)^2 = 4$, true, so $(3,-2) \in R$. Check $(-2,1)$: $-2 \leq 1^2 = 1$, true, so $(-2,1) \in R$. But $(3,1)$: $3 \leq 1^2 = 1$ is false, so $(3,1) \notin R \Rightarrow$ **not transitive**.

Hence R is neither reflexive, nor symmetric, nor transitive.

Question 3

Check whether the relation R defined on the set {1,2,3,4,5,6} as R = {(a,b): b = a+1} is reflexive, symmetric or transitive.

Solution:

R = {(1,2), (2,3), (3,4), (4,5), (5,6)}

Reflexive: (1,1) needs 1 = 1+1 = 2, false $\Rightarrow$ **not reflexive**.

Symmetric: $(1,2) \in R$ but (2,1) needs 1 = 2+1 = 3, false $\Rightarrow$ **not symmetric**.

Transitive: $(1,2) \in R$ and $(2,3) \in R$, but (1,3) needs 3 = 1+1 = 2, false, so $(1,3) \notin R \Rightarrow$ **not transitive**.

Hence R is neither reflexive, nor symmetric, nor transitive.

Question 4

Show that the relation R in R (real numbers), defined as $R = \{(a,b): a \le b\}$, is reflexive and transitive but not symmetric.

Solution:

Reflexive: For every real a, $a \le a$ is true $\Rightarrow$ **reflexive**.
Symmetric: Take a = 1, b = 2. $1 \le 2$ is true, so $(1,2) \in R$. But $2 \le 1$ is false, so $(2,1) \notin R \Rightarrow$ **not symmetric**.
Transitive: If $a \le b$ and $b \le c$, then $a \le c$ (standard order property of real numbers) $\Rightarrow$ **transitive**.

Hence R is reflexive and transitive but not symmetric.

Question 5

Check whether the relation R in R defined by $R = \{(a,b): a \le b^3\}$ is reflexive, symmetric or transitive.

Solution:

Reflexive: Take a = 1/2. $a \le a^3$ means $1/2 \le 1/8$, false $\Rightarrow$ **not reflexive**.
Symmetric: Take a = 1, b = 2. $1 \le 2^3 = 8$, true, so $(1,2) \in R$. But $2 \le 1^3 = 1$ is false $\Rightarrow$ **not symmetric**.
Transitive: Take a = 3, b = 1.5, c = 1.2. Check $a \le b^3$: $3 \le (1.5)^3 = 3.375$, true. Check $b \le c^3$: $1.5 \le (1.2)^3 = 1.728$, true. But $a \le c^3$: $3 \le (1.2)^3 = 1.728$ is false $\Rightarrow$ **not transitive**.

Hence R is neither reflexive, nor symmetric, nor transitive.

Question 6

Show that the relation R in the set {1,2,3} given by R = {(1,2),(2,1)} is symmetric but neither reflexive nor transitive.

Solution:

Reflexive: (1,1), (2,2), (3,3) are not in R $\Rightarrow$ **not reflexive**.
Symmetric: $(1,2) \in R$ and $(2,1) \in R$ — the pair and its reverse are both present $\Rightarrow$ **symmetric**.
Transitive: $(1,2) \in R$ and $(2,1) \in R$, so we need $(1,1) \in R$, but it is not $\Rightarrow$ **not transitive**.

Hence R is symmetric but neither reflexive nor transitive.

Question 7

Show that the relation R in the set A of all books in a library of a college, given by R = {(x,y): x and y have the same number of pages}, is an equivalence relation.

Solution:

Reflexive: Book x has the same number of pages as itself $\Rightarrow (x,x) \in R \Rightarrow$ **reflexive**.
Symmetric: If x has the same number of pages as y, then y has the same number of pages as x $\Rightarrow$ **symmetric**.
Transitive: If x and y have the same page count, and y and z have the same page count, then x and z have the same page count $\Rightarrow$ **transitive**.

Since R is reflexive, symmetric and transitive, R is an equivalence relation.

Question 8

Show that the relation R in the set A = {1,2,3,4,5} given by $R = \{(a,b): |a-b| \text{ is even}\}$ is an equivalence relation. Show also that all the elements of {1,3,5} are related to each other, all the elements of {2,4} are related to each other, but no element of {1,3,5} is related to any element of {2,4}.

Solution:

Reflexive: $|a-a| = 0$, which is even $\Rightarrow (a,a) \in R$ for all a $\Rightarrow$ **reflexive**.

Symmetric: $|a-b| = |b-a|$, so if one is even so is the other $\Rightarrow$ **symmetric**.

Transitive: If $|a-b|$ is even and $|b-c|$ is even, then a and b have the same parity, and b and c have the same parity, so a and c have the same parity, making $|a-c|$ even $\Rightarrow$ **transitive**.

So R is an equivalence relation.

Elements of {1,3,5}: $|1-3|=2$, $|3-5|=2$, $|1-5|=4$ — all even, so every pair from {1,3,5} is related.

Elements of {2,4}: $|2-4| = 2$, even, so 2 and 4 are related.

Across the sets: e.g. $|1-2| = 1$, $|3-4| = 1$, $|5-2| = 3$ — all odd, so no element of {1,3,5} is related to any element of {2,4} (odd and even numbers always differ by an odd amount).

Hence R is an equivalence relation, with {1,3,5} and {2,4} as separate related groups.

Question 9

Show that the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by $R = \{(a,b): |a-b| \text{ is a multiple of } 4\}$, is an equivalence relation. Find the set of all elements related to 1.

Solution:

$A = \{0,1,2,...,12\}$.

Reflexive: $|a-a| = 0$ is a multiple of 4 ($0 = 4\times0$) $\Rightarrow$ **reflexive**.

Symmetric: $|a-b| = |b-a|$, so if one is a multiple of 4, so is the other $\Rightarrow$ **symmetric**.

Transitive: If $a-b = 4k$ and $b-c = 4m$ for integers k,m, then $a-c = (a-b)+(b-c) = 4(k+m)$, a multiple of 4 $\Rightarrow$ **transitive**.

So R is an equivalence relation.

Elements related to 1: we need x in A with $|x-1|$ a multiple of 4, i.e. $x-1 \in \{0, \pm4, \pm8, \pm12\}$, giving $x \in \{1, 5, 9, 13, -3, -7, -11\}$. Keeping only values in $A = \{0,...,12\}$:

The set of elements related to 1 is {1, 5, 9}.

Question 10

Give an example of a relation which is: (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symmetric and transitive but not reflexive.

Solution:

Let $A = \{1,2,3\}$ throughout.

(i) $R = \{(1,2),(2,1)\}$. Symmetric (both pairs present); not reflexive ((1,1) missing); not transitive ($(1,2),(2,1) \in R$ but $(1,1) \notin R$).

(ii) $R = \{(1,2)\}$. Transitive (vacuously — no chain of two pairs exists to test); not reflexive ((1,1) missing); not symmetric ((2,1) missing).

(iii) $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$. Reflexive (all (x,x) present); symmetric (each pair's reverse is present); not transitive, since $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$.

(iv) R = {(a,b): $a \le b$} on real numbers (or R = {(1,1),(2,2),(3,3),(1,2),(1,3),(2,3)} on A). Reflexive and transitive as shown in Q4, but not symmetric since $(1,2) \in R$ while $(2,1) \notin R$.

(v) R = {(1,1),(2,2),(1,2),(2,1)} on A (note (3,3) excluded). Symmetric (pairs and reverses present); transitive ((1,2),(2,1) gives $(1,1) \in R$; (2,1),(1,2) gives $(2,2) \in R$); not reflexive since $(3,3) \notin R$.

Question 11

Show that the relation R in the set A of points in a plane given by R = {(P,Q): distance of P from the origin is the same as the distance of Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0,0)$ is the circle passing through P with the origin as centre.

Solution:

Reflexive: distance of P from origin = distance of P from origin $\Rightarrow (P,P) \in R \Rightarrow$ **reflexive**.
Symmetric: if dist(P) = dist(Q), then dist(Q) = dist(P) $\Rightarrow$ **symmetric**.
Transitive: if dist(P) = dist(Q) and dist(Q) = dist(S), then dist(P) = dist(S) $\Rightarrow$ **transitive**.

So R is an equivalence relation.

For a fixed point P (P ≠ origin) at distance r from the origin, every point Q related to P must also be at distance r from the origin. The set of all such points is exactly the set of points at a fixed distance r from the origin.

This set of points is precisely the circle of radius r centred at the origin, which passes through P.

Question 12

Show that the relation R defined in the set A of all triangles as R = {(T1,T2): T1 is similar to T2}, is equivalence relation. Consider three right angle triangles T1 with sides 3, 4, 5; T2 with sides 5, 12, 13 and T3 with sides 6, 8, 10. Which triangles among T1, T2 and T3 are related?

Solution:

Reflexive: Every triangle is similar to itself $\Rightarrow$ **reflexive**.
Symmetric: If T1 is similar to T2, then T2 is similar to T1 $\Rightarrow$ **symmetric**.
Transitive: If T1 is similar to T2, and T2 is similar to T3, then T1 is similar to T3 $\Rightarrow$ **transitive**.

So R is an equivalence relation.

Checking T1, T2, T3: Compare corresponding sides ratios.
T1 and T3: 3/6 = 1/2, 4/8 = 1/2, 5/10 = 1/2 — all ratios equal, so T1 and T3 are similar.
T1 and T2: 3/5, 4/12 = 1/3, 5/13 — not all equal, so T1 and T2 are not similar.
T2 and T3: 5/6, 12/8 = 3/2, 13/10 — not all equal, so T2 and T3 are not similar.

Hence T1 is related to T3, but neither is related to T2.

Question 13

Show that the relation R defined in the set A of all polygons as R = {(P1,P2): P1 and P2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Solution:

Reflexive: A polygon has the same number of sides as itself ⇒ **reflexive**.
Symmetric: If P1 and P2 have the same number of sides, so do P2 and P1 ⇒ **symmetric**.
Transitive: If P1, P2 have the same number of sides, and P2, P3 have the same number of sides, then P1, P3 have the same number of sides ⇒ **transitive**.

So R is an equivalence relation.

The triangle T has 3 sides, so every polygon related to T must also have exactly 3 sides.

The set of elements related to T is the set of all triangles in A.

Question 14

Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L1,L2): L1 is parallel to L2}. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Solution:

Reflexive: Every line is parallel to itself ⇒ **reflexive**.
Symmetric: If L1 is parallel to L2, then L2 is parallel to L1 ⇒ **symmetric**.
Transitive: If L1 is parallel to L2 and L2 is parallel to L3, then L1 is parallel to L3 ⇒ **transitive**.

So R is an equivalence relation.

The line $y = 2x + 4$ has slope 2. Any line parallel to it must also have slope 2 and can have any y-intercept c.

The set of lines related to $y = 2x + 4$ is $\{y = 2x + c : c$ is a real number$\}$.

Question 15

Let R be the relation in the set {1,2,3,4} given by R = {(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)}. Choose the correct answer: (A) R is reflexive and symmetric but not transitive (B) R is reflexive and transitive but not symmetric (C) R is symmetric and transitive but not reflexive (D) R is an equivalence relation.

Solution:

Reflexive: (1,1), (2,2), (3,3), (4,4) are all present in R ⇒ **reflexive**.
Symmetric: $(1,2) \in R$ but $(2,1) \notin R$ ⇒ **not symmetric**.
Transitive: Check all possible chains: (1,3),(3,2) ⇒ $(1,2) \in R$ ✓; (1,3),(3,3) ⇒ $(1,3) \in R$ ✓; (3,2),(2,2) ⇒ $(3,2) \in R$ ✓; (1,2),(2,2) ⇒ $(1,2) \in R$ ✓. All chains hold ⇒ **transitive**.

Correct answer: (B) R is reflexive and transitive but not symmetric.

Question 16

Let R be the relation in the set N given by R = {(a,b): a = b – 2, b > 6}. Choose the correct answer: (A) $(2,4) \in R$ (B) $(3,8) \in R$ (C) $(6,8) \in R$ (D) $(8,7) \in R$.

Solution:

Check each option using both conditions $a = b–2$ and $b > 6$:
(2,4): $b–2 = 2 = a$ ✓, but $b = 4$, not > 6 ✗ — rejected.
(3,8): $b–2 = 6 \neq 3$ ✗ — rejected.
(6,8): $b–2 = 6 = a$ ✓, and $b = 8 > 6$ ✓ — satisfies both conditions.
(8,7): $b–2 = 5 \neq 8$ ✗ — rejected.

Correct answer: (C) $(6,8) \in R$.

End of Exercise 1.1. Exercises 1.2, 1.3, 1.4 and the Miscellaneous Exercise for Chapter 1 will follow in subsequent parts, followed by Chapters 2–13.