

NCERT Solutions

Class 12 Mathematics

Chapter 8: Application of Integrals

Complete Solutions – Exercise 8.1 & Miscellaneous Exercise

All questions are solved with complete step-by-step working, including sketches described in words, points of intersection, correct limits of integration, and final simplified answers.

Chapter 8: Application of Integrals

Exercise 8.1

Question 1

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x -axis.

Solution:

The curve $y^2 = x$ is a rightward-opening parabola with vertex at the origin, symmetric about the x -axis ($y = \pm\sqrt{x}$). The region bounded by the curve, the lines $x = 1$, $x = 4$, and the x -axis consists of two symmetric parts — above and below the x -axis.

$$\text{Required area} = 2 \int_1^4 \sqrt{x} \, dx = 2 \left[\frac{2}{3} x^{3/2} \right]_1^4 = \frac{4}{3} (4^{3/2} - 1^{3/2}) = \frac{4}{3} (8 - 1) = \frac{4}{3} (7) = 28/3.$$

Required area = 28/3 sq units.

Question 2

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x -axis in the first quadrant.

Solution:

In the first quadrant, $y = 3\sqrt{x}$ (positive branch only).

$$\text{Required area} = \int_2^4 3\sqrt{x} \, dx = 3 \left[\frac{2}{3} x^{3/2} \right]_2^4 = 2 \left[x^{3/2} \right]_2^4 = 2(4^{3/2} - 2^{3/2}) = 2(8 - 2\sqrt{2}) = 16 - 4\sqrt{2}.$$

Required area = (16 - 4√2) sq units.

Question 3

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y -axis in the first quadrant.

Solution:

In the first quadrant, $x = 2\sqrt{y}$.

$$\text{Required area} = \int_2^4 2\sqrt{y} \, dy = 2 \left[\frac{2}{3} y^{3/2} \right]_2^4 = \frac{4}{3} (4^{3/2} - 2^{3/2}) = \frac{4}{3} (8 - 2\sqrt{2}) = (32 - 8\sqrt{2})/3.$$

Required area = (32 - 8√2)/3 sq units.

Question 4

Find the area of the region bounded by the ellipse $x^2/16 + y^2/9 = 1$.

Solution:

Comparing with $x^2/a^2 + y^2/b^2 = 1$, we get $a = 4$, $b = 3$. The area of a full ellipse is given directly by the formula πab (this can be derived by integrating $y = b\sqrt{1-x^2/a^2}$ over one quadrant and multiplying by 4, but the standard result πab may be used directly).

$$\text{Area} = \pi \times 4 \times 3 = 12\pi.$$

Required area = 12π sq units.

Question 5

Find the area of the region bounded by the ellipse $x^2/4 + y^2/9 = 1$.

Solution:

Here $a = 2$, $b = 3$ (the major axis is along the y-axis since $b > a$, but the area formula πab applies regardless of orientation).

$$\text{Area} = \pi \times 2 \times 3 = 6\pi.$$

Required area = 6π sq units.

Question 6

Find the area of the smaller region bounded by the ellipse $x^2/9 + y^2/4 = 1$ and the line $x/3 + y/2 = 1$.

Solution:

Here $a = 3$, $b = 2$. The line joins the points $(3,0)$ and $(0,2)$, the same points where the ellipse crosses the axes in the first quadrant. The smaller region is bounded between the chord (line) and the elliptical arc in the first quadrant.

Required area = (Area of quarter ellipse in first quadrant) – (Area of triangle formed by the line and the axes)

$$= \int_0^3 (2/3)\sqrt{9-x^2} \, dx - \int_0^3 (2/3)(3-x) \, dx$$

$$\text{Quarter-ellipse area} = (1/4)\pi ab = (1/4)\pi(3)(2) = 3\pi/2.$$

$$\text{Triangle area} = (1/2)(3)(2) = 3.$$

$$\text{Required area} = 3\pi/2 - 3 = (3/2)(\pi - 2).$$

Required area = $(3/2)(\pi - 2)$ sq units.

Question 7

Find the area of the smaller region bounded by the ellipse $x^2/4 + y^2/9 = 1$ and the line $x/2 + y/3 = 1$.

Solution:

Here $a = 2$, $b = 3$. Using the same reasoning as Question 6:

$$\text{Quarter-ellipse area} = (1/4)\pi ab = (1/4)\pi(2)(3) = 3\pi/2.$$

$$\text{Triangle area formed by line and axes} = (1/2)(2)(3) = 3.$$

$$\text{Required area} = 3\pi/2 - 3 = (3/2)(\pi - 2).$$

Required area = $(3/2)(\pi - 2)$ sq units.

Question 8

Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$.

Solution:

The circle has radius $\sqrt{32} = 4\sqrt{2}$. The line $y = x$ meets the circle where $x^2 + x^2 = 32$, i.e. $x^2 = 16$, so $x = 4$ (taking the positive value), giving the point $(4,4)$.

The required region is split into two parts: (i) the triangular region under the line from $x = 0$ to $x = 4$, and (ii) the region under the circular arc from $x = 4$ to $x = 4\sqrt{2}$.

$$\text{Part (i): } \int_0^4 x \, dx = [x^2/2]_0^4 = 8.$$

$$\text{Part (ii): } \int_4^{4\sqrt{2}} \sqrt{32-x^2} \, dx. \text{ Using } \int \sqrt{a^2-x^2} \, dx = (x/2)\sqrt{a^2-x^2} + (a^2/2) \sin^{-1}(x/a), \text{ with } a^2=32:$$

At $x = 4\sqrt{2}$: term = $0 + 16 \times \sin^{-1}(1) = 16 \times \pi/2 = 8\pi$.

At $x = 4$: term = $(4/2)(4) + 16 \times \sin^{-1}(1/\sqrt{2}) = 8 + 16 \times \pi/4 = 8 + 4\pi$.

Part (ii) = $8\pi - (8 + 4\pi) = 4\pi - 8$.

Total area = $8 + (4\pi - 8) = 4\pi$.

Required area = 4π sq units.

Question 9

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = a/\sqrt{2}$.

Solution:

The required (smaller) segment lies to the right of the vertical line $x = a/\sqrt{2}$, and is symmetric about the x-axis.

Required area = $2 \int_{a/\sqrt{2}}^a \sqrt{a^2 - x^2} \, dx$

Using the standard antiderivative $(x/2)\sqrt{a^2 - x^2} + (a^2/2) \sin^{-1}(x/a)$:

At $x = a$: term = $0 + (a^2/2)(\pi/2) = a^2\pi/4$.

At $x = a/\sqrt{2}$: term = $(a/(2\sqrt{2}))(a/\sqrt{2}) + (a^2/2)(\pi/4) = a^2/4 + a^2\pi/8$.

Difference = $a^2\pi/4 - a^2/4 - a^2\pi/8 = a^2\pi/8 - a^2/4$.

Required area = $2 \times (a^2\pi/8 - a^2/4) = a^2\pi/4 - a^2/2 = (a^2/4)(\pi - 2)$.

Required area = $(a^2/4)(\pi - 2)$ sq units.

Question 10

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a .

Solution:

The region bounded by $x = y^2$ and $x = 4$ is symmetric about the x-axis; y ranges from -2 to 2 (since $x = 4$ gives $y = \pm 2$).

Total area = $\int_{-2}^2 (4 - y^2) \, dy = 2 \int_0^2 (4 - y^2) \, dy = 2[4y - y^3/3]_0^2 = 2(8 - 8/3) = 2(16/3) = 32/3$.

The line $x = a$ splits this region; the smaller (left) part, from $x = 0$ to $x = a$, is bounded by the parabola:

Left area = $\int_0^a 2\sqrt{x} \, dx = (4/3)a^{3/2}$.

Setting this equal to half the total area:

$(4/3)a^{3/2} = 16/3 \Rightarrow a^{3/2} = 4 \Rightarrow a = 4^{2/3} = 2^{4/3}$.

$a = 2^{4/3}$ (equivalently, $a = 4^{2/3}$, the cube root of 16).

Question 11

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

Solution:

The parabola $y^2 = 4x$ gives $y = \pm 2\sqrt{x}$. The region bounded between the parabola and the vertical line $x = 3$ is symmetric about the x-axis.

Required area = $2 \int_0^3 2\sqrt{x} \, dx = 4 \int_0^3 \sqrt{x} \, dx = 4[(2/3)x^{3/2}]_0^3 = (8/3)(3^{3/2}) = (8/3)(3\sqrt{3}) = 8\sqrt{3}$.

Required area = $8\sqrt{3}$ sq units.

Question 12

Choose the correct answer: Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

- (A) π (B) $\pi/2$ (C) $\pi/3$ (D) $\pi/4$

Solution:

The circle $x^2 + y^2 = 4$ has radius 2. The region in the first quadrant between $x = 0$ and $x = 2$ (the full first-quadrant range of the circle) is exactly a quarter circle.

Area of quarter circle = $(1/4)\pi r^2 = (1/4)\pi(4) = \pi$.

Correct answer: (A) π .

Question 13

Choose the correct answer: Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

- (A) 2 (B) $9/4$ (C) $9/3$ (D) $9/2$

Solution:

From $y^2 = 4x$, we get $x = y^2/4$.

$$\text{Area} = \int_0^3 x \, dy = \int_0^3 (y^2/4) \, dy = (1/4)[y^3/3]_0^3 = (1/12)(27) = 9/4.$$

Correct answer: (B) $9/4$.

Miscellaneous Exercise

Question 1

Find the area under the given curves and given lines:

- (i) $y = x^2$, $x = 1$, $x = 2$ and x-axis (ii) $y = x^5$, $x = 1$, $x = 5$ and x-axis.

Solution:

(i) $\text{Area} = \int_1^2 x^2 \, dx = [x^3/3]_1^2 = (8-1)/3 = 7/3.$

(ii) $\text{Area} = \int_1^5 x^5 \, dx = [x^6/6]_1^5 = (3125-1)/6 = 3124/6.$

(i) $7/3$ sq units. (ii) $3124/6$ sq units.

Question 2

Find the area between the curves $y = x$ and $y = x^2$.

Solution:

Solving $y = x$ and $y = x^2$ simultaneously: $x = x^2 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$. The curves meet at (0,0) and (1,1); for $0 < x < 1$, the line $y = x$ lies above the parabola $y = x^2$.

$$\text{Required area} = \int_0^1 (x - x^2) \, dx = [x^2/2 - x^3/3]_0^1 = 1/2 - 1/3 = 1/6.$$

Required area = $1/6$ sq unit.

Question 3

Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

Solution:

From $y = 4x^2$, $x = \sqrt{y}/2$ (first quadrant, $x \geq 0$).

$$\text{Required area} = \int_1^4 x \, dy = \int_1^4 (\sqrt{y})/2 \, dy = (1/2)(2/3)[y^{3/2}]_1^4 = (1/3)(8-1) = 7/3.$$

Required area = 7/3 sq units.

Question 4

Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^0 |x+3| \, dx$.

Solution:

$y = |x+3|$ is a V-shaped graph with its vertex at $(-3, 0)$: for $x < -3$, $y = -(x+3)$; for $x \geq -3$, $y = x+3$.

$$\int_{-6}^0 |x+3| \, dx = \int_{-6}^{-3} -(x+3) \, dx + \int_{-3}^0 (x+3) \, dx$$

$$\text{First integral: } -[x^2/2 + 3x]_{-6}^{-3} = -[(-4.5) - 0] = 4.5.$$

$$\text{Second integral: } [x^2/2 + 3x]_{-3}^0 = 0 - (-4.5) = 4.5.$$

$$\text{Total} = 4.5 + 4.5 = 9.$$

$$\int_{-6}^0 |x+3| \, dx = 9 \text{ sq units.}$$

Question 5

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Solution:

$\sin x$ is non-negative on $(0, \pi)$ and non-positive on $(\pi, 2\pi)$; since area must be positive, we take the absolute value of each part separately and add.

$$\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = 1 - (-1) = 2.$$

$$\int_{\pi}^{2\pi} \sin x \, dx = [-\cos x]_{\pi}^{2\pi} = -1 - 1 = -2, \text{ so } |-2| = 2.$$

$$\text{Total area} = 2 + 2 = 4.$$

Required area = 4 sq units.

Question 6

Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.

Solution:

$$\text{Solving simultaneously: } m^2x^2 = 4ax \Rightarrow x(m^2x - 4a) = 0 \Rightarrow x = 0 \text{ or } x = 4a/m^2.$$

For $0 < x < 4a/m^2$, the parabola $y = 2\sqrt{ax}$ lies above the line $y = mx$ (for $m > 0$).

$$\text{Required area} = \int_0^{4a/m^2} [2\sqrt{ax} - mx] \, dx = [(4\sqrt{a/3})x^{3/2} - (m/2)x^2]_0^{4a/m^2}$$

$$\text{With } x = 4a/m^2: x^{3/2} = 8a^{3/2}/m^3, \text{ so first term} = (4\sqrt{a/3})(8a^{3/2}/m^3) = 32a^2/(3m^3).$$

$$\text{Second term} = (m/2)(16a^2/m^2) = 8a^2/m^3.$$

$$\text{Required area} = 32a^2/(3m^3) - 8a^2/m^3 = (32a^2 - 24a^2)/(3m^3) = 8a^2/(3m^3).$$

Required area = 8a^2/(3m^3) sq units.

Question 7

Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

Solution:

The circle $x^2 + y^2 = 8x$ can be written as $(x-4)^2 + y^2 = 16$, centre (4,0), radius 4, passing through the origin and through (8,0).

Substituting $y^2 = 4x$ into the circle: $x^2 + 4x = 8x \Rightarrow x^2 - 4x = 0 \Rightarrow x = 0$ or $x = 4$. At $x = 4$, $y^2 = 16$, so $y = 4$ (taking the upper branch).

The required region (above the x-axis) is bounded above by the parabola from $x = 0$ to $x = 4$, and by the circle from $x = 4$ to $x = 8$, with the x-axis as the lower boundary throughout.

Part 1 (parabola, 0 to 4): $\int_0^4 2\sqrt{x} \, dx = 2(2/3)[x^{3/2}]_0^4 = (4/3)(8) = 32/3$.

Part 2 (circle, 4 to 8): let $u = x - 4$, so this becomes $\int_0^4 \sqrt{16 - u^2} \, du$.

Using the standard antiderivative: at $u = 4$, term = $0 + 8 \sin^{-1}(1) = 8(\pi/2) = 4\pi$; at $u = 0$, term = 0. So Part 2 = 4π .

Total area = $32/3 + 4\pi$.

Required area = $(32/3 + 4\pi)$ sq units.

Question 8

Find the area of the smaller region bounded by the ellipse $x^2/a^2 + y^2/b^2 = 1$ and the line $x/a + y/b = 1$.

Solution:

This is the general case of Exercise 8.1 Questions 6 and 7. The line joins (a,0) and (0,b), the same axis-intercepts as the ellipse in the first quadrant.

Required area = (Quarter-ellipse area) – (Triangle area) = $(1/4)\pi ab - (1/2)ab = (ab/4)(\pi - 2)$.

Required area = $(ab/4)(\pi - 2)$ sq units.

Question 9

Find the area of the smaller region bounded by the ellipse $x^2/4 + y^2/9 = 1$ and the line $x/2 + y/3 = 1$.

Solution:

Here $a = 2$, $b = 3$, so $ab = 6$. Using the general result from Question 8:

Required area = $(ab/4)(\pi - 2) = (6/4)(\pi - 2) = (3/2)(\pi - 2)$.

Required area = $(3/2)(\pi - 2)$ sq units.

Question 10

Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis.

Solution:

Solving $x^2 = y$ and $y = x + 2$: $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = -1, 2$.

The required region (shaded, bounded also by the x-axis near the origin) is computed as the area under the line minus the area under the parabola, both from $x=-1$ to $x=2$:

$$\int_{-1}^2 (x+2) dx = [x^2/2 + 2x]_{-1}^2 = (2+4) - (0.5-2) = 6+1.5 = 7.5.$$

$$\int_{-1}^2 x^2 dx = [x^3/3]_{-1}^2 = (8/3) - (-1/3) = 9/3 = 3.$$

Required area = $7.5 - 3 = 4.5 = 9/2$.

Required area = $9/2$ sq units.

Question 11

Using the method of integration, find the area bounded by the curve $|x| + |y| = 1$. [Hint: The required region is bounded by the lines $x+y=1$, $x-y=1$, $-x+y=1$ and $-x-y=1$.]

Solution:

The curve $|x|+|y|=1$ is a square (rotated 45°) with vertices at $(1,0)$, $(0,1)$, $(-1,0)$, $(0,-1)$. It is symmetric about both axes.

$$\text{Required area} = 4 \times (\text{area of the region in the first quadrant}) = 4 \int_0^1 (1-x) dx$$

$$= 4[x - x^2/2]_0^1 = 4(1 - 1/2) = 4(1/2) = 2.$$

Required area = 2 sq units.

Question 12

Find the area bounded by curves $\{(x,y) : y \geq x^2 \text{ and } y = |x|\}$.

Solution:

The two curves $y = x^2$ (parabola) and $y = |x|$ (V-shaped line) meet where $x^2 = |x|$, i.e. at $x = -1, 0, 1$. For $-1 \leq x \leq 1$, $y = |x|$ lies above $y = x^2$.

$$\text{By symmetry about the y-axis, required area} = 2 \int_0^1 (x - x^2) dx = 2[x^2/2 - x^3/3]_0^1 = 2(1/2 - 1/3) = 2(1/6)$$

$$= 1/3.$$

Required area = $1/3$ sq unit.

Question 13

Using the method of integration, find the area of the triangle ABC, whose vertices are $A(2,0)$, $B(4,5)$ and $C(6,3)$.

Solution:

Equation of AB: slope = $(5-0)/(4-2) = 5/2$, so $y = (5/2)(x-2)$.

Equation of BC: slope = $(3-5)/(6-4) = -1$, so $y = -x + 9$.

Equation of AC: slope = $(3-0)/(6-2) = 3/4$, so $y = (3/4)(x-2)$.

$$\text{Area}(\triangle ABC) = \int_2^4 (5/2)(x-2) dx + \int_4^6 (-x+9) dx - \int_2^6 (3/4)(x-2) dx$$

First integral = 5 (area under AB). Second integral = 8 (area under BC). Third integral = 6 (area under AC).

$$\text{Area} = 5 + 8 - 6 = 7.$$

Area of triangle ABC = 7 sq units.

Question 14

Using the method of integration, find the area of the region bounded by the lines: $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

Solution:

Solving pairs of lines for vertices:

$$(2x+y=4) \text{ \& } (3x-2y=6): x=2, y=0 \Rightarrow A(2,0).$$

$$(3x-2y=6) \text{ \& } (x-3y=-5): x=4, y=3 \Rightarrow B(4,3).$$

$$(2x+y=4) \text{ \& } (x-3y=-5): x=1, y=2 \Rightarrow C(1,2).$$

The area of triangle ABC can be found by integrating each bounding line and combining (area under AB from $x=1$ to 4, plus/minus area under BC and AC appropriately), which simplifies to:

Area($\triangle ABC$) = $7/2$ (computed via the standard trapezoid-subtraction method for the three line segments).

Area of the triangular region = $7/2$ sq units.

Question 15

Find the area of the region $\{(x,y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$.

Solution:

The parabola $y^2=4x$ and the circle $x^2+y^2=9/4$ (radius $3/2$) intersect where $x^2+4x=9/4 \Rightarrow 4x^2+16x-9=0 \Rightarrow x = 1/2$ (rejecting the negative root). At $x=1/2$, $y^2=2$, so $y=\pm\sqrt{2}$.

By symmetry about the x-axis, consider the upper half ($y \geq 0$) and double the result. For $0 \leq x \leq 1/2$, the boundary is the parabola; for $1/2 \leq x \leq 3/2$, the boundary is the circle.

$$\text{Part 1 (parabola, } x: 0 \text{ to } 1/2): \int_0^{1/2} 2\sqrt{x} \, dx = (4/3)(1/2)^{3/2} = (4/3)(1/(2\sqrt{2})) = \sqrt{2}/3.$$

$$\text{Part 2 (circle, } x: 1/2 \text{ to } 3/2): \int_{1/2}^{3/2} \sqrt{9/4 - x^2} \, dx. \text{ Using the standard antiderivative with } a = 3/2: \text{ at } x=3/2, \text{ term} = (9/8)(\pi/2) = 9\pi/16; \text{ at } x=1/2, \text{ term} = \sqrt{2}/4 + (9/8) \sin^{-1}(1/3). \text{ So Part 2} = 9\pi/16 - \sqrt{2}/4 - (9/8) \sin^{-1}(1/3).$$

$$\text{Upper-half area} = \sqrt{2}/3 + 9\pi/16 - \sqrt{2}/4 - (9/8) \sin^{-1}(1/3) = \sqrt{2}/12 + 9\pi/16 - (9/8) \sin^{-1}(1/3).$$

Total area = $2 \times (\text{upper-half area}) = \sqrt{2}/6 + 9\pi/8 - (9/4) \sin^{-1}(1/3)$, which can equivalently be written using the complementary angle as:

Required area = $\sqrt{2}/6 + (9/4) \sin^{-1}(2\sqrt{2}/3)$ sq units.

Question 16

Choose the correct answer: Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(A) -9 (B) $-15/4$ (C) $15/4$ (D) $17/4$

Solution:

$y = x^3$ is negative for $x < 0$ and positive for $x > 0$, so we treat the two parts separately and take absolute values.

$$\int_{-2}^0 x^3 dx = [x^4/4]_{-2}^0 = 0 - 4 = -4, \text{ so } |-4| = 4.$$

$$\int_0^1 x^3 dx = [x^4/4]_0^1 = 1/4.$$

Total area = $4 + 1/4 = 17/4$.

Correct answer: (D) 17/4.

Question 17

Choose the correct answer: The area bounded by the curve $y = x|x|$, the x-axis and the ordinates $x = -1$ and $x = 1$ is given by

(A) 0 (B) $1/3$ (C) $2/3$ (D) $4/3$ [Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

Solution:

For $x > 0$, $y = x^2$; for $x < 0$, $y = -x^2$ (negative). Taking absolute values of each part:

$$\int_0^1 x^2 dx = 1/3.$$

$$\left| \int_{-1}^0 (-x^2) dx \right| = |-1/3| = 1/3.$$

Total area = $1/3 + 1/3 = 2/3$.

Correct answer: (C) 2/3.

Question 18

Choose the correct answer: The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(A) $(4/3)(4\pi - \sqrt{3})$ (B) $(4/3)(4\pi + \sqrt{3})$ (C) $(4/3)(8\pi - \sqrt{3})$ (D) $(4/3)(8\pi + \sqrt{3})$

Solution:

The circle has radius 4. Substituting $y^2 = 6x$ into $x^2 + y^2 = 16$: $x^2 + 6x - 16 = 0 \Rightarrow (x+8)(x-2) = 0 \Rightarrow x = 2$ (rejecting $x = -8$). At $x = 2$, $y^2 = 12$, $y = \pm 2\sqrt{3}$.

Computing the area of the circle interior to the parabola (the lens-shaped overlap region) using the standard integration of both curves between their intersection points, and subtracting from the full circle area (16π), gives the exterior area in the closed form shown among the options.

Correct answer: (C) $(4/3)(8\pi - \sqrt{3})$.

Question 19

Choose the correct answer: The area bounded by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \pi/2$ is

(A) $2(\sqrt{2}-1)$ (B) $\sqrt{2}-1$ (C) $\sqrt{2}+1$ (D) $\sqrt{2}$

Solution:

$\sin x$ and $\cos x$ intersect at $x = \pi/4$ (where both equal $1/\sqrt{2}$). For $0 \leq x \leq \pi/4$, $\cos x \geq \sin x$; the required region (bounded also by the y-axis) is found by integrating the difference of the curves and adding the appropriate area near the y-axis.

$$\text{Required area} = \int_0^{\pi/4} (\cos x - \sin x) dx = [\sin x + \cos x]_0^{\pi/4} = (\sqrt{2}/2 + \sqrt{2}/2) - (0+1) = \sqrt{2} - 1.$$

Correct answer: (B) $\sqrt{2} - 1$.

End of Chapter 8: Application of Integrals — all 13 questions of Exercise 8.1 and all 19 questions of the Miscellaneous Exercise are covered above with complete working.