Class 12 Maths Formulas

Chapter-wise formula sheet for CBSE Class 12 | Quick revision with conditions

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1 Relations and Functions

Counting (|A| = m, |B| = n)

Relations from A to B	2^{mn}
Relations on a set of n elements	2^{n^2}
Functions from A to B	n^m
One-one functions from A to B	$\dfrac{n!}{(n-m)!} \quad (n \geq m);\ \ 0 \text{ if } n < m$
Bijections (when m = n)	$n!$

Types of relations and functions

Equivalence relation	reflexive, symmetric and transitive
One-one (injective)	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
Onto (surjective)	Range of f = Codomain of f
Bijective	one-one and onto

Composition and inverse

Composite function	$(g \circ f)(x) = g\big(f(x)\big)$
Non-commutative	$g \circ f \neq f \circ g$ (in general)
Inverse function	f^{-1} exists $\iff$ f is bijective
Inverse of a composite	$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Condition: g∘f is defined only when the range of f lies within the domain of g.

2 Inverse Trigonometric Functions

Remember: $\sin^{-1} x$ means "the angle whose sine is x". It is not $\dfrac{1}{\sin x}$, which is $\operatorname{cosec} x$.

Principal value branches

Function	Domain	Principal value range
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	$\mathbb{R}$	$(0, \pi)$

Function	Domain	Principal value range
$\sec^{-1} x$	$\mathbb{R} - (-1, 1)$	$[0,\, \pi] - \{\frac{\pi}{2}\}$
$\operatorname{cosec}^{-1} x$	$\mathbb{R} - (-1, 1)$	$[-\frac{\pi}{2},\, \frac{\pi}{2}] - \{0\}$

Standard identities

$\sin^{-1}(-x) = -\sin^{-1} x$	$\tan^{-1}(-x) = -\tan^{-1} x$
$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$	$\cos^{-1}(-x) = \pi - \cos^{-1} x$
$\cot^{-1}(-x) = \pi - \cot^{-1} x$	$\sec^{-1}(-x) = \pi - \sec^{-1} x$
$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \quad x \in [-1, 1]$	$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in \mathbb{R}$
$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, \quad \lvert x\rvert \geq 1$	$\sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} x, \quad \lvert x\rvert \geq 1$
$\cos^{-1} \frac{1}{x} = \sec^{-1} x, \quad \lvert x\rvert \geq 1$	$\tan^{-1} \frac{1}{x} = \cot^{-1} x, \quad x > 0$

Sum, difference and double-angle forms

Sum	$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \dfrac{x + y}{1 - xy}, \quad xy < 1$
Difference	$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \dfrac{x - y}{1 + xy}, \quad xy > -1$
Double angle (sine form)	$2\tan^{-1} x = \sin^{-1} \dfrac{2x}{1 + x^2}, \quad \lvert x\rvert \leq 1$
Double angle (cosine form)	$2\tan^{-1} x = \cos^{-1} \dfrac{1 - x^2}{1 + x^2}, \quad x \geq 0$
Double angle (tangent form)	$2\tan^{-1} x = \tan^{-1} \dfrac{2x}{1 - x^2}, \quad -1 < x < 1$

Caution: $\sin^{-1}(\sin x) = x$ only when $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. For other x, first bring the angle into the principal range.

3 Matrices

Operations and properties

Matrix product	$(AB)_{ij} = \sum_{k} a_{ik}\, b_{kj}$
Order of product	$A_{m\times n}\, B_{n\times p} = C_{m\times p}$
Not commutative	$AB \neq BA \;\; \text{(in general)}$
Associative, distributive	$(AB)C = A(BC), \quad A(B + C) = AB + AC$
Identity	$AI = IA = A$
Zero product	$AB = O \;\not\Rightarrow\; A = O \text{ or } B = O$

Transpose

$(A^T)^T = A$	$(A + B)^T = A^T + B^T$
$(kA)^T = kA^T$	$(AB)^T = B^T A^T$

Symmetric and skew-symmetric matrices

Symmetric	$A^T = A$

Skew-symmetric	$A^T = -A$ (all diagonal entries are 0)
Every square matrix	$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Condition: The product AB is defined only when the number of columns of A equals the number of rows of B.

4 Determinants

Remember: A matrix is an arrangement of numbers. Its determinant is a single number, and it exists only for a square matrix.

Evaluation

2 × 2 determinant	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
3 × 3 (along first row)	$a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$
Area of a triangle	$\Delta = \frac{1}{2}\left\|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right\|$
Collinear points	determinant above $= 0$

Properties (order n)

$\|A^T\| = \|A\|$	$\|AB\| = \|A\|\,\|B\|$
$\|kA\| = k^n\|A\|$	$\|\operatorname{adj} A\| = \|A\|^{n-1}$

- Interchanging two rows (or columns) changes the sign of the determinant.
- If two rows (or columns) are identical or proportional, the determinant is 0.
- Multiplying one row (or column) by k multiplies the determinant by k.
- Adding a multiple of one row to another row leaves the determinant unchanged.

Minors, cofactors, adjoint and inverse

Cofactor	$A_{ij} = (-1)^{i+j} M_{ij} \quad (M_{ij} = \text{minor})$
Expansion	$\|A\| = \sum_j a_{ij} A_{ij}$ (any one row or column)
Adjoint	$\operatorname{adj} A =$ transpose of the cofactor matrix
Key relation	$A\,(\operatorname{adj} A) = (\operatorname{adj} A)\,A = \|A\|\,I$
Inverse	$A^{-1} = \frac{1}{\|A\|}\operatorname{adj} A$
Reversal laws	$(AB)^{-1} = B^{-1}A^{-1}, \quad (A^T)^{-1} = (A^{-1})^T$

Condition: A^{-1} exists if and only if $|A| \neq 0$ (A is non-singular).

System of equations AX = B

If $\|A\| \neq 0$	unique solution $X = A^{-1}B$
If $\|A\| = 0$ and $(\text{adj } A)B \neq O$	no solution (inconsistent)
If $\|A\| = 0$ and $(\text{adj } A)B = O$	infinitely many solutions or none

5 Continuity and Differentiability

Continuity at x = c	$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$
Derivative at x = c	$f'(c) = \lim_{h\to 0} \dfrac{f(c+h)-f(c)}{h}$

Remember: Every differentiable function is continuous, but a continuous function need not be differentiable. For example, |x| is not differentiable at 0.

Standard derivatives

$\dfrac{d}{dx}(x^n) = n\,x^{n-1}$	$\dfrac{d}{dx}(e^x) = e^x$
$\dfrac{d}{dx}(a^x) = a^x \ln a, \quad a>0$	$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}, \quad x>0$
$\dfrac{d}{dx}(\sin x) = \cos x$	$\dfrac{d}{dx}(\cos x) = -\sin x$
$\dfrac{d}{dx}(\tan x) = \sec^2 x$	$\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
$\dfrac{d}{dx}(\sec x) = \sec x \tan x$	$\dfrac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Derivatives of inverse trigonometric functions

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}, \quad \lvert x\rvert<1$	$\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}, \quad \lvert x\rvert<1$
$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$	$\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1+x^2}$
$\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}, \quad \lvert x\rvert>1$	$\dfrac{d}{dx}(\operatorname{cosec}^{-1} x) = -\dfrac{1}{\lvert x\rvert\sqrt{x^2-1}}, \quad \lvert x\rvert>1$

Rules of differentiation

Product rule	$(uv)' = u'v + uv'$
Quotient rule	$\left(\dfrac{u}{v}\right)' = \dfrac{u'v-uv'}{v^2}, \quad v\neq 0$
Chain rule	$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx}$
Logarithmic differentiation	$y=u^v,\ u>0: \quad \dfrac{dy}{dx} = u^v\left[\dfrac{v}{u}u' + v'\ln u\right]$
Parametric form	$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}, \quad \dfrac{dx}{dt}\neq 0$
Second derivative (parametric)	$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$

- Implicit differentiation: differentiate both sides with respect to x, treating y as a function of x, then solve for dy/dx.

6 Applications of Derivatives

Rate of change	$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$
Increasing function	$f'(x) > 0$ on an interval
Decreasing function	$f'(x) < 0$ on an interval
Tangent at (x_0, y_0)	$y - y_0 = f'(x_0)\,(x - x_0)$
Normal at (x_0, y_0)	$y - y_0 = -\frac{1}{f'(x_0)}\,(x - x_0), \quad f'(x_0) \neq 0$
Approximation	$f(x + \Delta x) \approx f(x) + f'(x)\,\Delta x$

Maxima and minima

Critical points	$f'(x) = 0$ or $f'(x)$ does not exist
First derivative test	f' changes $(+) \to (-)$: local max; $(-) \to (+)$: local min
Second derivative test	$f'(c) = 0$: $f''(c) < 0 \Rightarrow$ local max; $f''(c) > 0 \Rightarrow$ local min

Condition: f′(c) = 0 is a necessary condition for a local maximum or minimum at an interior point c, not a sufficient one. If f″(c) = 0, the second derivative test fails; use the first derivative test.

- Absolute maximum/minimum on [a, b]: compare f at the critical points inside the interval and at the end points a and b.

7 Integrals

Note: Add the constant of integration C to every indefinite integral below.

Basic formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\int e^x dx = e^x + C$	$\int a^x dx = \frac{a^x}{\ln a} + C$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$	$\int \operatorname{cosec} x\,dx = \ln\lvert\operatorname{cosec} x - \cot x\rvert + C$

Standard forms

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

Methods of integration

Substitution	$t = g(x): \quad \int f(g(x))\,g'(x)\,dx = \int f(t)\,dt$
Logarithmic form	$\int \frac{f'(x)}{f(x)}\,dx = \ln\lvert f(x)\rvert + C$
By parts	$\int u\,v\,dx = u\int v\,dx - \int\left(u'\int v\,dx\right)dx$
Special case	$\int e^x\,[\,f(x)+f'(x)\,]\,dx = e^x f(x) + C$
Partial fractions	$\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$
Repeated factor	$\frac{1}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{D}{x-b}$

- For by parts, choose u by ILATE: Inverse trigonometric, Logarithmic, Algebraic, Trigonometric, Exponential.

Definite integrals

Fundamental theorem	$\int_a^b f(x)\,dx = F(b)-F(a), \quad F'(x)=f(x)$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f = \int_a^c f + \int_c^b f, \quad a<c<b$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & f \text{ even} \\ 0, & f \text{ odd} \end{cases}$$

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & f(2a-x)=f(x) \\ 0, & f(2a-x)=-f(x) \end{cases}$$

8 Applications of Integrals

Area under y = f(x)	$A = \int_a^b y\,dx = \int_a^b f(x)\,dx \quad (\text{between } x=a,\ x=b)$
Area along the y-axis	$A = \int_c^d x\,dy = \int_c^d g(y)\,dy \quad (\text{between } y=c,\ y=d)$
Area between two curves	$A = \int_a^b \big[f(x)-g(x)\big]dx, \quad f(x)\ge g(x) \text{ on } [a,b]$

Circle $x^2 + y^2 = a^2$	$\text{Area} = \pi a^2$
Ellipse $x^2/a^2 + y^2/b^2 = 1$	$\text{Area} = \pi ab$

Condition: Find the limits by solving f(x) = g(x). Always sketch the region first. If the curves cross inside the interval, split the integral at the point of intersection. Take the area of any part below the x-axis as positive, so that it does not cancel the part above.

9 Differential Equations

Order	order of the highest-order derivative
Degree	power of the highest-order derivative

- Degree is defined only when the differential equation is a polynomial in its derivatives.

Number of constants	general solution has as many arbitrary constants as the order

Variable separable

Form	$\dfrac{dy}{dx} = f(x)\,g(y)$
Solution	$\displaystyle\int \frac{dy}{g(y)} = \int f(x)\,dx + C, \quad g(y) \neq 0$

Homogeneous equation

Form	$\dfrac{dy}{dx} = F(x,y), \quad F(\lambda x, \lambda y) = \lambda^0 F(x,y)$
Substitution	$y = vx \;\Rightarrow\; \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ (for $\dfrac{dx}{dy}$ form, put $x = vy$)

Linear equation of first order

In y	$\dfrac{dy}{dx} + Py = Q, \quad P, Q$ functions of x
Integrating factor	$\text{I.F.} = e^{\int P\,dx}$
Solution	$y \cdot \text{I.F.} = \displaystyle\int Q \cdot \text{I.F.}\,dx + C$
In x	$\dfrac{dx}{dy} + P_1 x = Q_1 : \quad x \cdot e^{\int P_1 dy} = \displaystyle\int Q_1\, e^{\int P_1 dy}\,dy + C$

10 Vector Algebra

Basics

Position vector of P(x, y, z)	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \quad \lvert\vec{r}\rvert = \sqrt{x^2+y^2+z^2}$
Vector AB	$\overrightarrow{AB} = (\text{position vector of B}) - (\text{position vector of A})$
Unit vector along a	$\hat{a} = \dfrac{\vec{a}}{\lvert\vec{a}\rvert}$
Direction cosines of a	$l = \dfrac{a_1}{\lvert\vec{a}\rvert},\ m = \dfrac{a_2}{\lvert\vec{a}\rvert},\ n = \dfrac{a_3}{\lvert\vec{a}\rvert}; \quad l^2 + m^2 + n^2 = 1$
Section formula (internal, m : n)	$\dfrac{m\vec{b} + n\vec{a}}{m+n}$
Section formula (external, m : n)	$\dfrac{m\vec{b} - n\vec{a}}{m-n}$

Mid-point of A and B	$\frac{\vec{a}+\vec{b}}{2}$

Scalar (dot) product

Definition	$\vec{a}\cdot\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
Angle between vectors	$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$
Square of magnitude	$\vec{a}\cdot\vec{a} = \lvert\vec{a}\rvert^2$
Perpendicular vectors	$\vec{a}\perp\vec{b} \iff \vec{a}\cdot\vec{b} = 0 \quad (\text{non-zero vectors})$
Projection of a on b	$\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$
Vector component of a along b	$\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert^2}\vec{b}$

Vector (cross) product

Definition	$\vec{a}\times\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta\,\hat{n}, \quad \lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta$
Determinant form	$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$
Anti-commutative	$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
Parallel vectors	$\vec{a}\parallel\vec{b} \iff \vec{a}\times\vec{b} = \vec{0} \quad (\text{non-zero vectors})$
Unit vector ⊥ to both	$\pm\frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert}$
Area of triangle (sides a, b)	$\frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$
Area of parallelogram (sides a, b)	$\lvert\vec{a}\times\vec{b}\rvert$
Area of parallelogram (diagonals d_1, d_2)	$\frac{1}{2}\lvert\vec{d_1}\times\vec{d_2}\rvert$

- Unit vector products: $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$, and $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$.

11 Three-Dimensional Geometry

Direction cosines and direction ratios

Direction cosines	$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma; \quad l^2 + m^2 + n^2 = 1$
From direction ratios a, b, c	$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}$
Line through two points	$\text{D.R.'s} = x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$

Equation of a line

Vector form (point a, parallel to b)	$\vec{r} = \vec{a} + \lambda\vec{b}$
Cartesian form	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
Through two points	$\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a});\ \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Angle between two lines

Vector form	$\cos\theta = \dfrac{\lvert\vec{b}_1 \cdot \vec{b}_2\rvert}{\lvert\vec{b}_1\rvert\lvert\vec{b}_2\rvert}$
Cartesian form (D.R.'s)	$\cos\theta = \dfrac{\lvert a_1a_2 + b_1b_2 + c_1c_2\rvert}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$
Perpendicular lines	$a_1a_2 + b_1b_2 + c_1c_2 = 0$
Parallel lines	$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Shortest distance

Skew lines (vector form)	$d = \dfrac{\left\lvert(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)\right\rvert}{\lvert\vec{b}_1 \times \vec{b}_2\rvert}$
Skew lines (Cartesian form)	$d = \dfrac{\left\lVert\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}\right\rVert}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$
Parallel lines	$d = \dfrac{\left\lvert\vec{b}\times(\vec{a}_2-\vec{a}_1)\right\rvert}{\lvert\vec{b}\rvert}$

- If the shortest distance between two lines is 0, the lines intersect.

12 Linear Programming

Objective function	$Z = ax + by$ (to be maximised or minimised)
Constraints	linear inequalities, with $x \geq 0,\ y \geq 0$

- The feasible region is the common region satisfying all constraints. Every point in it is a feasible solution.

Corner point method

- 1. Draw the constraints and identify the feasible region.
- 2. Find the coordinates of every corner point of the feasible region.
- 3. Evaluate Z at each corner point. Let M be the largest and m the smallest value.

Condition: Bounded region: both the maximum and the minimum exist, and each occurs at a corner point. Unbounded region: M is the maximum only if the open half-plane ax + by > M has no point in common with the feasible region; m is the minimum only if ax + by < m has no point in common with it. Otherwise, there is no maximum or minimum. If two corner points give the same optimal value, every point on the segment joining them is also optimal. If there is no feasible region, the problem has no solution.

13 Probability

Conditional probability

Definition	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)},\quad P(B) \neq 0$
Complement	$P(A' \mid B) = 1 - P(A \mid B)$

Multiplication theorem

Two events	$P(A \cap B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$

Three events	$P(A \cap B \cap C) = P(A)\,P(B \mid A)\,P(C \mid A \cap B)$

Independent events

Condition	$P(A \cap B) = P(A)\,P(B)$

- If A and B are independent, then so are A and B′, A′ and B, and A′ and B′.
- Three events are mutually independent only if every pair is independent and P(A ∩ B ∩ C) = P(A) P(B) P(C).

Condition: Independent events are not the same as mutually exclusive events. Two events with non-zero probabilities cannot be both.

Total probability and Bayes' theorem

- Let $E_1, E_2, \ldots, E_n$ be mutually exclusive and exhaustive events with $P(E_i) > 0$, and let A be any event.

Total probability theorem	$P(A) = \sum_{i=1}^{n} P(E_i)\,P(A \mid E_i)$
Bayes' theorem	$P(E_i \mid A) = \dfrac{P(E_i)\,P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A \mid E_j)}, \quad P(A) > 0$

Always check the conditions attached to a formula before using it. Confirm chapter scope against your current CBSE syllabus and NCERT textbook.